THE·PRINCIPLES·OF STRUCTURAL MECHANICS

PERCY J. WALDRAM

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THE PRINCIPLES OF STRUCTURAL MECHANICS

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TREATED WITHOUT THE USE OF HIGHER MATHEMATICS

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PREFACE

IN view of the large number of text-books already published on Structural Mechanics, and of the comprehensive character of many of them, a word of explanation is due upon the appearance of yet another addition to the list.

It is common experience that Architectural draughtsmen, although acquainted with numerous mathematical and graphic methods of solving *straightforward* problems in structural design, are frequently nonplussed by slight complications, or even by a small variation from the form in which these problems are accustomed to present themselves. Also that when required to justify the calculation of any given dimensions they are at best usually only able to quote printed authority from the formula or graphic method adopted, and are unable to give any further reason for having used it.

Similar proof of dangerously superficial knowledge is to be found—to a lesser extent certainly, but far more frequently than is at all desirable—in the work of young engineers whose studies have covered the wide range which is now customary; and one is forced to the conclusion that the study of first principles has hitherto either been treated in too brief and cursory a manner or presented in a way which has not succeeded in carrying conviction to the mind of the average student.

A long but barren search for a simple text-book which would obviate the necessity of explaining to draughtsmen the construction and genesis of those formulæ which are now in common use, has led the author to the conclusion that a book which deals thoroughly with first principles and which explains not only the application but also the construction of ordinary formulæ would not be without value to the student anxious to acquire a thorough and practical knowledge.

At the same time he ventures to hope that engineers, architects, and others who have to deal with structural problems at infrequent intervals may find of use a book which does not demand a revision of one's knowledge of higher mathematics.

The author desires to express his thanks to the Editor and Publisher of the *Builder* for permission to reproduce part of his series of articles which appeared in that journal under the title of "Some Notes upon Definitions used in Structural Mechanics"; to the Editor and Publishers of the *Architect's and Builder's Journal* for permission to reproduce an article upon "Wind Pressure"; to Messrs Blackwood; and also to the publishers of the *London Magazine* for permission to publish Miss Macleod's photographs of African domical huts; to the Council of the Junior Institution of Engineers for permission to reproduce an extract from a contribution to their Transactions dealing with the effect of wind pressure on sloping roofs; and to Dr Stanton of the National Physical Laboratory for the preparation of condensed tables of the results of his extensive experiments on that subject. The author has utilised the invaluable reseaches of Professor T. Claxton Fidler in dealing with the theory of deflection and columns, and those of Mr William Dunn on the theory of Domes.

In a book necessarily written and revised in the scanty leisure afforded by a professional practice, faults and shortcomings are probably inevitable, and the author would welcome suggestions for rendering it more useful. He has endeavoured to include such examples of the practical calculation of structural problems as are necessary to illustrate briefly the use of the rules and formulæ dealt with; but it was found that the inclusion of fully worked out calculations of typical problems of practical design with proper working drawings in sufficient number to be commensurate with the scope of ordinary structural design, would have rendered the book inconveniently bulky and its cost prohibitive to many who might otherwise find it useful. Such practical calculations are of great value both to the student and the designer, provided they are calculated on a basis which is either familiar to the reader or which is co-ordinated with some book to which he can refer. It is therefore hoped that a considerable number may be produced as a companion work at an early date.

With the same object of keeping down size and cost, several chapters dealing with the principles of design in reinforced concrete have similarly been relegated to a separate volume.

PERCY J. WALDRAM.

12 Buckingham Street,
Charing Cross, W.C.,
March 1912.

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NOTATION

UNLESS otherwise stated, the symbols used have the following meaning:—

A	Superficial area of cross section.
BM	Bending moment.
b	Breadth in inches.
c.g.	Centre of gravity.
D	Depth in feet.
d	Depth in inches.
E	Modulus of elasticity.
f	Fibre stress.
f_0	Modulus of rupture. Ultimate fibre stress under bending.
H	Height in feet.
h	Height in inches.
I	Moment of inertia.
L	Length in feet.
l	Length in inches.
MR	Moment of resistance.
r	Radius of gyration.
S	Span in feet.
t	Thickness in inches.
W	Total load.
w	Load per unit of length.
x y	Distance of extreme fibre from neutral axis.
Z	Section modulus.
$+$	Sign of addition. Compressive stress.
$-$	Sign of subtraction. Tensile stress.
$\propto$	Varies as.
↻ ↺	Clockwise and anti-clockwise moments.

PART I

LEVERAGE AND GRAPHIC STATICS

CHAPTER I

INTRODUCTION

Danger of Superficial Knowledge—Standard Rules and Formulæ only Applicable to Simple Problems—Danger of Over Confidence in Formulæ and of Undue Contempt for Theory—Advantage of Thorough Knowledge — Advantage of Experimental Investigation — Effect of Sound Structural Knowledge upon Design—Natural Forces Unchanged with Changes in Construction—Incomplete Knowledge no Excuse after an Accident.

DANGER OF SUPERFICIAL KNOWLEDGE.—The subject of structural mechanics is one which lends itself very readily to mathematical and geometrical solutions, and these can be expressed in the highly condensed form of standard rules and formulæ for dealing with direct and straightforward problems.

The student is thereby very frequently induced to attempt to cover a wide field by committing to memory a large number of rules, without inquiring too closely into their origin and the principles and data upon which they are founded. Some countenance is lent to this by those writers of text-books who have been similarly tempted to convey a large amount of information concentrated into a small compass by either omitting altogether the tedious explanations of how each formula and geometrical rule is built up step by step, or by stating those explanations in the briefest mathematical form. Unfortunately, mathematical analysis, like any other delicate instrument, is liable to rust and deteriorate rapidly when not kept in constant use and adjustment; and the busy student who wishes if possible to satisfy himself as to the correctness of a formula, and the harassed designer who wishes to vary it for correct application to a given case, are often alike unsatisfied.

It cannot be stated too strongly that nowadays the subject of structural mechanics is one in which a little knowledge is a most particularly dangerous thing. Frank ignorance is much safer than a smattering of half-understood formulæ and rules learnt in parrot fashion. Those who undertake the design of buildings for modern requirements may at any time have to incur great and even terrible responsibilities—responsibilities of far greater moment than consideration of appearance, proportions, or cost—which can only be met by a clear first-hand knowledge of the principles of stresses. *Natural forces cannot be controlled by formulæ*, but they can be and always have been controlled, since man first began to build, by the applications of the principles which formulæ briefly express.

STANDARD RULES AND FORMULÆ ONLY APPLICABLE TO SIMPLE PROBLEMS.—The glamour of the rapid acquisition of knowledge often blinds the student and the young designer to the point upon which nearly all text-books are discreetly silent, viz., that general rules and formulæ, however accurate and trustworthy, are applicable only to problems of a direct and simple nature. It happens only too frequently that he discovers how seldom the structural problems of modern building are either direct or simple only when many other matters are demanding his attention, and when there is little time, and perhaps less inclination, for further study and research.

Even when a problem is apparently straightforward and approximately simple, it is almost invariably complicated by such conflicting considerations as stability and lightness, market sizes and restricted dimensions; appearance, durability, and cost; and calls for something considerably more than the superficial knowledge acquired by the wholesale assimilation of standard rules.

DANGER OF OVER CONFIDENCE IN FORMULÆ AND OF UNDUE CONTEMPT FOR THEORY.—The ease and rapidity with which formulæ and graphic geometrical methods solve difficult but straightforward problems often invest them with a highly fictitious value, so that instead of being used merely as more or less convenient and practical expressions of the action of forces and reactions, they are regarded almost with reverence, and accepted as mysterious scientific talismans by which natural forces may be controlled to the use and convenience of man.

Sometimes this undeserved confidence is rapidly and rudely dispelled by some accident arising from the indiscriminate use of formulæ. Sometimes the process of disillusion is effected more slowly by a series of disappointments over the shortcomings of formulæ and rules when applied to the determination of practical everyday problems. But—speedy or tardy—it is almost invariably followed by a strong and permanent reaction against "theory" of all kinds and a preference for rule-of-thumb methods and for copying the dimensions of existing work of proved stability. In the hands of men possessing both experience and constructive ability, such approximations may be, and for work of ordinary character frequently are, both safe and fairly economical. But in the young designer they are apt to stifle all originality, and to lead to great and even dangerous waste of material. Quite as many failures have been caused by the dead weight of unnecessary material put in so as to be "well on the safe side" as can be attributed to unduly slender proportions.

ADVANTAGE OF THOROUGH KNOWLEDGE.—It is impossible to know too much and difficult to know enough about any rule or formula used to determine safe dimensions. In its brief mathematical form it frequently partakes of the character of a "one job" tool, like a pair of compasses fixed at one particular radius. But when its fundamental principles have been thoroughly grasped, its construction followed out in detail, and its various constants and data appraised at their true practical value, then it becomes a useful instrument. It can be adjusted to meet variations in material and complications in requirements—its capabilities as well as its limitations are known to the designer, and it can be used with safe discretion—neither credited with imaginary powers nor mistrusted with equally ignorant contempt. Then and only then is it worthy of that confidence which is unfortunately only too frequently bestowed upon any combination of letters and symbols possessing the authority of printer's ink.

The student who perseveres in the apparently tedious process of building up all his knowledge from first principles is rapidly rewarded by finding that this somewhat dry subject is instinct with a life and interest which is largely obscured by the more rapid mathematical treatment ; and he also finds his labours considerably lightened by the fact that the simple principles of leverage and reaction give him the key to practically all ordinary solutions.

An attempt has been made in the following pages to show the steps by which all ordinary methods of calculating dimensions are thus built up. It is not suggested that every solution used in practice has been analysed, but it will be found that those which can be thus dealt with by the aid of mathematics of the simplest character form a fairly comprehensive list, and include many subjects, such as deflection, the theory of the funicular polygon, dome stresses, etc., which are usually described, even in advanced text-books, as being too complicated for explanation.

ADVANTAGE OF EXPERIMENTAL INVESTIGATION.—The student's work will gain both in interest and value if he can learn his principles and prove their application by even the simplest experiments. A practical test will teach him more in five minutes than can be assimilated from hours of study, and such tests can often be made quite as accurately with materials costing a few pence as with delicate apparatus costing as many pounds. Amongst the apparatus for experiments which are described herein as being such as can be profitably carried out by classes of students, will be found many which cost only a nominal sum for materials and which can be easily constructed by anyone possessing a few tools and a little spare time. Both the time and the money would be well spent, for a principle learnt by experiment is learnt both thoroughly and permanently, whilst one's conception of a principle which is arrived at only by intellectual deduction is apt in time to become hazy and unreliable.

A good text-book may do much to make the subject clear, but its very clearness is apt to relieve the student of the necessity for digging his practical knowledge out of mathematical obscurity ; although probably not one student in a hundred will have sufficient patience to properly perform this useful but dreary task. Private experimental work will teach a great deal, but to most men its range is extremely limited. The author's experience tends to show that by far the best method of thoroughly learning the subject consists of a complete course of experimental research, involving the actual measurement of forces, stresses, and strains by the student, coupled with clear explanations, and frequent and well-supervised exercise in practical design.

The value of experimental work is thoroughly appreciated in the training of engineers. It is largely neglected, but is even more desirable, in the teaching of building construction and in the training of architects, who have not the engineers' advantage of being called upon to constantly exercise their mathematical knowledge.

EFFECT OF SOUND STRUCTURAL KNOWLEDGE UPON DESIGN.—It is a common fallacy amongst architects that the science of structural mechanics is a very uninteresting department of engineering work which has been unjustly imposed upon them by the necessity of modern building. In the first place, it is not necessarily an uninteresting subject, neither is it one which is at all alien to true architecture.

The ability to produce good architectural design is not, of course, conferred by good structural knowledge ; but defective structural knowledge almost invariably mars design which is otherwise good.

Engineers certainly often design ugly structures, but that is not because they know too much of stresses and strains, but because they know too little —because their work is bad engineering. Material exactly applied to its work without weak parts and without excess possesses the vital elements of true beauty, whereas material unfitted for its obvious work is always, to that extent, ugly. It does no harm to a sense of artistic proportion to learn how to place material exactly where it is required ; on the contrary, it generally does it a great deal of good.

Some of the greatest triumphs of Gothic architecture form a veritable living picture of thrust and counter-thrust, and it is more than probable that the first introduction of the pointed arch was due to the structural weakness of the semicircular Roman form which it superseded. If Brunelleschi, Bramante, Wren, and the great minds of the Renaissance were not ashamed to consider themselves engineers first, and artists afterwards, the modern architect, who has to design in steel and reinforced concrete, as well as in wood, stone, and brick, can hardly afford to relegate his structural problems entirely to engineering specialists.

Perhaps the criteria of good design most sacred to educated architects are the proportions of classic Greek buildings ; but all these proportions, the ratio of diameter to height of columns, of cornices, of the thickness of piers and pilasters, etc., were originally derived from the accumulated experience of countless generations of craftsmen who were largely guided by the utilitarian considerations of strength, durability, and economy, which are now so often considered to be foreign to true art.

NATURAL FORCES UNCHANGED WITH CHANGES IN CONSTRUCTION.—It is well to bear in mind that there is nothing new either in these considerations or in the natural forces to which all structures are exposed. The forces of gravity, wind pressure, compression, and tension have not altered since the introduction of mild steel and reinforced concrete. The forces which broke down the Tay Bridge were precisely the same as those which were encountered by the first savage who piled stones to make walls and covered them with tree boughs and skins. We may know more about these forces, and being able to resist them with better materials, we may dare to challenge them more boldly and with greater skill—but they are always the same. What we imprison in the steel frame skyscraper of to-day are precisely the same forces which were imprisoned in the first log hut, and they stress the latest reinforced concrete jetty precisely as they stressed the first pile-supported lake dwelling.

INCOMPLETE KNOWLEDGE NO EXCUSE AFTER AN ACCIDENT.—It cannot be too strongly impressed upon the student that those natural forces, when they are underrated or neglected, are as dangerous as ever, and can break out and punish ignorance or carelessness as terribly as they have always done. We are so accustomed to the apparently inert solidity of safe structures that we are very apt to overlook the fact that forces which lie hidden in every part have potentialities of destruction to life and property, and are only silent and invisible because they have been carefully weighted down and curbed. When, however, an accident happens, and heavy materials do begin to move and "take charge" it is seldom indeed that some lives are not lost. Then it is useless for the designer to try and hide behind the authority of a standard text-book; he must bear the consequences of the responsibility he has undertaken.

If a heavily-loaded beam buckles at the ends under the action of horizontal shear, and not at the centre where failure was expected and provided against, it is no consolation to the widow and orphans of a workman who happened to be underneath that the designer's text-book had never pointed out the object of web stiffeners. He ought to have known about it ere he called himself a designer at all.

It is just as idle to think that natural forces can be controlled by half-understood formulæ as it would be to suppose that the art of warfare could be acquired by reading up text-books on strategy. The soldier must learn completely the principles of attack, defence, and tactics which underlie the broad principles of strategy, or run the risk of paying for his ignorance with his life. The designer who considers himself capable, on the strength of having read about a few formulæ, of battling with the unsleeping and unresting forces of nature runs the risk of paying for his ignorance with the lives of his fellow men. Of the two the soldier's hazard is perhaps preferable.

CHAPTER II

FORCE AND REACTION

Force—Three Effects of Force—Reaction—The Nature of Simple Tension and Compression—At least Two Forces necessary to Produce any kind of Stress—Inertia—Stress and Strain—Various Forms of Stresses and their Corresponding Strains—Measurement of Stress and Strain.

BEFORE proceeding to examine forces and resistances in detail, it is necessary to clearly understand some elementary facts which are more or less opposed to popular ideas, and to define certain terms to which an unusual meaning is attached in connection with this subject. Other similar terms can be more conveniently defined when occasion arises to consider that which they indicate.

FORCE.—Force can be measured by its effect upon matter. What it is we do not know, we are only concerned here with its effects, viz. :—

THREE EFFECTS OF FORCE.—

(1) Imparting motion to bodies, or accelerating, retarding, or changing the motion of bodies.

(2) Opposing or balancing other known forces.

(3) Producing a visible deflection, distortion, or, as it is called, a "strain," in ductile bodies.

The subject of structural mechanics, dealing as it does mainly with statics, or the science of still bodies, is not largely concerned with the first effect, which is chiefly of importance in the study of "dynamics" or in the science of moving bodies. Everyday examples of it are easy to find. A cricket ball is bowled, the motion being imparted to it by the muscular force of the bowler. It meets the muscular force of the batsman, and its motion is resisted or changed. A coin is spun up in the air, motion being imparted to it by the muscular force of the hand. The force of gravity acts upon it, reducing the rate of its upward movement until it stops and reverses it.

The third effect, *i.e.*, the ability of a force to deflect or distort ductile bodies, can be as easily grasped. If a weight be hung upon a spring balance the spring is deflected or distorted. The force of gravity acting upon the weight can by this means be recognised and its magnitude measured. Or a beam can be loaded, and the force of gravity can be recognised in the deflection which it causes ; and if the deflection can be accurately measured, the effect of the force of gravity (in other words the load) upon the beam can be measured. Many other examples will readily suggest themselves.

The second effect by which we recognise the action of a force, viz., its ability to balance and counteract other forces, is obviously of the utmost importance in the study of still bodies and structures, in which all forces, both internal or external, permanent or occasional, must of necessity be balanced.

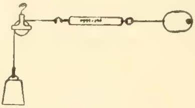


Fig. 1.

It is not difficult to conceive of two opposing forces balancing each other. Fig. 1 shows a pin loaded with a weight exercising a sideways pull which can be measured by the distortion of

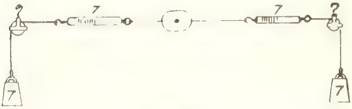


FIG. 2.

the spring in a spring balance introduced into the fine silk cord carrying the weight. Fig. 2 shows the same pin obviously relieved from the sideways pull by the action of an equal weight. A great deal can be learned from this simple experiment. If the pulleys are well made, the effect of placing a small coin on one or other of the weights will be to upset the state of balance and cause a tendency to movement in the direction of the greater weight. This is precisely what would be expected. Common experience leads us to suppose that when two unequal weights are opposed to each other, the weaker is always pushed or pulled in the direction in which the stronger force is moving, in the way that a strong team in a tug-of-war pulls along a weaker team.

The mind so naturally assumes that movement always takes place with unbalanced forces, that it is worth while to step somewhat out of the way in order to test the truth of that important assumption a little further. In Fig. 3 the two nearly frictionless pulleys are replaced with nails, and the fine silk cords with twine. It will now be found that even under decidedly unequal weights no movement takes place, and the assumption is apparently *proved* to be unreliable. This commonplace experiment is useful in showing

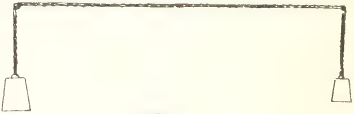


FIG. 3.

not only how easy it is to deduce incorrect conclusions from hasty observations, but it also shows how necessary it is to look out for forces whose activities may not be apparent. The fact that a force is not doing anything that can be seen, heard, or felt is no evidence that it is inert or negligible. In this case the hidden resistance of *Friction* is at work on behalf of the smaller weight, and by its silent but effective assistance is enabling it to balance the larger weight. Movement will only take place when such inequality exists that the difference between the two weights overcomes the friction between the string and the nails.

One of the most useful properties of practical experiments is that the mind is constantly trained to the habit of investigating the *whole* of the forces acting against a structure or likely to act against it. In fighting the forces of nature it is necessary to do a great deal of intelligent scouting. Scouting does not consist so much of finding the enemy and counting his exposed troops, as in ascertaining what probable places of cover are near his position, and then finding out whether, and if so, how, they are occupied. In structural mechanics every one of the forces likely to affect a structure must be located and accounted for. Anyone can recognise the active ones—they are seldom overlooked. It is those which are usually static which must be specially sought out — they have a way of unexpectedly showing extreme activity. This habit of mind—the natural healthy result of experimental study—should be most carefully fostered. It is generally the ambushes which lead to regrettable incidents ; it is generally the unexpected activity of neglected forces which lead to so-called accidents.

REACTION.—When the fact is satisfactorily established from these and similar experiments that forces tend to movement, and that when forces are truly balanced there is no motion, then the natural assumption follows that when an undoubtedly active force acts upon a body without causing movement, that force must of necessity be balanced by some other force whose activity may not be otherwise apparent. This assumption forms the basis of all static theory, but it is occasionally somewhat difficult to appreciate. Let the reader press his hand on to a desk or table. There will be no doubt as to the thrust acting against his hand. The muscles of his arm indicate it, the ductile texture of the hand is distorted; yet there is no movement. But the possibility of some hidden balancing force must not be forgotten, and in this case there is the very important one of *reaction*—a force which we seldom properly appreciate because we so seldom see it at work. It is one which it is necessary to become thoroughly well acquainted with as soon as possible. It is not easy to conceive—how can the desk push against the hand? The desk is dead, inert, and incapable of individual movement. Materials, however, are often by no means so inert and lifeless as they seem—in many of their characteristics they are almost human. If it be desired to see what activity material is capable of without movement as a whole, let a granite wall be violently struck with a heavy sledge hammer. The investigator will then have very little doubt indeed that the reaction of apparently inert material is a living force.

When a thrust is exerted upon any body or structure which remains stationary, it simply opposes it in the form of reaction.

It should be carefully noticed that material never hits back any harder than it is hit. If you kick a wall with a 5-lb. kick, you feel no more than a 5-lb.

reaction ; if you kick it with a 30-lb. kick, you receive almost a 30-lb. reaction. It may be quite capable of exerting a 300-lb. reaction, but it does not do so except under a 300-lb. provocation. It also hits back in very much the same manner that it is hit. If the thumb be pressed lightly and continuously upon the table, the table presses lightly and continuously back. If the table be struck with a sudden heavy blow with the fist, it returns a sudden and severe reaction.

Action and reaction are opposite in direction, and equal in amount.

Instances of the sudden active retaliation of material when struck with a sharp blow, although useful in illustrating reaction, do not afford much help towards appreciating the continuous reaction which takes place in structures. This can be better seen in the case of a weight, say 2 cwt., hanging by a rope secured to a beam. There is an obvious tension of 224 lbs. on the rope due to the weight, but there is no movement. What, then, is the balancing force? Obviously the beam is resting upon its supports, and they, having no use for 2 cwt. of downward pressure, return it, and are actually pulling the rope up. As, however, they cannot move upwards, it seems, as before, strange that they can pull upwards. If the investigator is still in doubt he should climb on to the beam and untie the rope ; his arms and hands will then soon tell him how much actual pulling the apparently dead beam and supports were capable of.

THE NATURE OF SIMPLE TENSION AND COMPRESSION.—Any idea that reaction is only an imaginary theoretical force invented by mathematicians in order to solve problems is erroneous, and must be mentally thrown overboard. It is now necessary to similarly jettison another ordinary conception as to the nature of simple tension and compression. The rope in the last example, when tied to the beam, was under a tension of 2 cwt., due to the weight. If it is *pulled* upwards by a muscular pull of 2 cwt., the tension on it would appear to be 4 cwt.

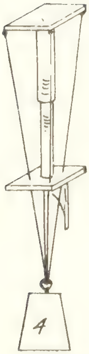


Fig. 4.

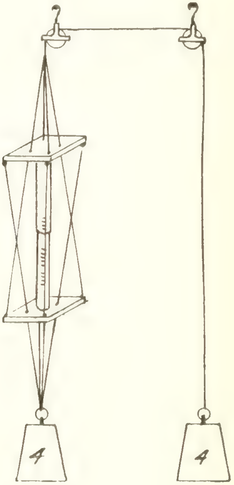


Fig. 5.

This point can easily be investigated with the simple apparatus shown in Fig. 1, in which a spring balance is loaded with a 7-lb. weight. Add another weight, as shown in Fig. 2, and it will be found that in spite of the two 7-lb. weights pulling on the balance the same tension is indicated in both cases.

Or, again, take the apparatus shown in Figs. 4 and 5.

When the 4-lb. weight is applied to drag down the top of the compression balance standing on a bracket (Fig. 4), the indicating needle points to 4 lbs., as would be expected. When another force of 4 lbs. is applied to pull up its base, as in Fig. 5, so that it can be lifted clear of the bracket, it still indicates a compression of only 4 lbs. and not 8 lbs.

If a man were to tie a rope to a post and pull on it with a 60-lb. pull, his back and muscles would be conscious of a 60-lb. effort. But if he were to unfasten it from the post and get another man to pull with a 60-lb. pull, his back and muscles would still only be conscious of the same effort as before. The tension on the rope is certainly not 120 lbs., because no one could pull that amount horizontally.

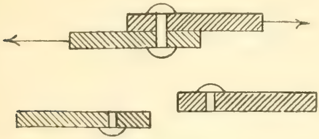
The real truth is that he could not obtain a 60-lb. tension on the rope at all without *two* equal and opposite pulls of 60 lbs. It would be useless to pick up the end of a loose coil of rope and try to exert a 60-lb. pull on it. It takes two to make a quarrel, and it takes two equal and opposite forces *or* reactions to make up what we understand as simple tension or simple compression; and until those two equal and opposite forces are present a tension or compression equal to one of them is impossible. In the absence of another man to pull, the *post* was called upon to supply a 60-lb. reaction, and the pin in Fig. 1 and the bracket in Fig. 4 act in a similar manner.

AT LEAST TWO FORCES NECESSARY TO PRODUCE ANY KIND OF STRESS.—A very little reflection will show that at least two forces (or a force and a reaction) are necessary in order to stress materials in *any* way. A piece of wood cannot be sawn through unless it be held or supported; a nut cannot be tightened on a bolt which is free to turn; and a rivet shank cannot be hammered into a head unless it be "held up."

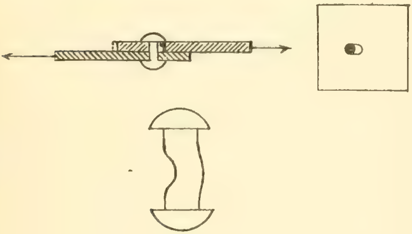
INERTIA.—The "inertia" of bodies, viz., their disinclination to movement, enables them to offer a reaction to suddenly applied forces, so that a small nail might be driven smartly into a heavy log freely suspended by a cord; and the end of a stick resting on a table for the greater part of its length, and covered only with a newspaper, may be snapped off by a smart blow. But in such cases the reaction, although transitory, is just as real as in those cases where it is permanent.

STRESS AND STRAIN.—The terms stress and strain are often popularly confused, but the student must be careful to keep their meanings distinct. Stress is the application of a pair of forces to material; strain is the *deformation* of the material which invariably results from the application of stress. Every material being more or less ductile or capable of distortion under stress, every form of stress produces its corresponding strain, and every strain has its characteristic form of failure, which generally varies with different materials.

VARIOUS FORMS OF STRESSES AND THEIR CORRESPONDING STRAINS.—Tension produces elongation, and eventually results in the tearing apart of the fibres with a reduction of cross sectional area at and near the point of fracture. Compression tends to produce lateral expansion, which causes brittle materials to splinter and break outwards, and ductile materials to bulge. Transverse stress causes bending, and failure occurs in the form of breaking across. Shearing stress causes a slip like the sliding of the top half of a pack of cards off the bottom half permitted by a rupture of the fibres on the face of the slip, as when a girder web cuts through the shank of a rivet (Fig. 6), or when a rivet hole is being punched in a plate. Bearing stress causes indentation, and failure occurs by a combination of crumpling and cutting, as when a strong rivet shank tears into and splits a plate through which it passes or *vice versa*. Torsion or twisting tends to wrench the fibres asunder by a combination of shear and tension.



Failure by Shearing.



Failure by Bearing.

Fig. 6.

Measurement of Stress and Strain.—Stress is measured with reference to the *area* of the cross section acted upon, as a tension of 5 tons *per square inch*, a compression of 5 tons *per square foot*, etc. Strain is measured by ascertaining by what proportion some original dimension of the stressed material has been varied. For instance a wrought-iron rod 1 in. by 1 in. by 10 ft. long under a tension of 5 tons, viz., under a *stress* of 5 tons *per square inch*, might suffer an elongation of $\frac{1}{20}$ in. This would be expressed as a fraction of the original length of 10 ft., or 120 in., viz., a strain of $\frac{1}{2400}$. If under a load of 1 ton the elongation were $\frac{1}{100}$ in., the strain would have been $\frac{1}{12000}$.

CHAPTER III

LOADS AND STRESSES

Live and Dead Load—Impact—Allowance for Shock and Vibration—Weight of Crowds—Superimposed Load or Super Load—Importance of Weight of Structure as an Element of the Total Load—Floor Loads—Behaviour of Materials under Stress—Permanent Set—Fatigue of Materials—Elastic Limit and Ultimate Stress—Factor of Safety.

LIVE AND DEAD LOAD.—The fact that a load suddenly applied will stress materials to a greater extent than one applied slowly or permanently is well known. The range of this enhanced effect can be clearly seen in an ordinary spring balance with a strip of brass wrapped round it, and nipped just tight enough to slide easily, but to remain in any position on the scale to which it may be pushed by the indicating needle. If a weight of say 4 lbs. be hung on such a balance, the spring is strained to an extent which causes the indicating needle to slide the brass strip down to the 4-lb. mark on the scale. Then let the weight be supported in some convenient way, so as to just bear on the hook without distorting the spring; if the support be then suddenly removed, and the weight allowed to act quickly on the spring, the brass strip will be found at the 8-lb. mark, although the needle will after a few vibrations finally come to rest as before at the 4-lb. mark. More exact experiments have established the fact that the effect of a load suddenly applied on any material is approximately double that of one slowly applied. The fibres of the material are, as it were, taken unawares, and not having time to collect their energies, are punished to *at least* double the extent they otherwise would be. For purposes of calculation such live loads are therefore usually doubled in order to obtain the equivalent dead loads.

IMPACT.—The force of impact must, if necessary, be allowed for in addition. If the 4-lb. weight were slung on a string loop and allowed to drop even a short distance, and thus gather momentum before acting on the hook, the brass slip would be found to register the same strain as 20 or 30 lbs. of dead load.

What may be classed as "engineering" structures, such as railway bridges, sea walls, lighthouses, etc., are largely subject to live load. But in ordinary buildings live loads have usually only to be allowed for on floors, balconies, or staircases liable to very sudden crowding, or which support heavy goods in rapid transit. It is as well, however, to remember that quite a slowly-moving load hauled over a small obstacle—like a cart wheel passing over a brick—may give rise to an impact which will induce greater stresses than those due to a live load.

ALLOWANCE FOR SHOCK AND VIBRATION.—In practical design the following allowances are sometimes made for shock and vibration. In buildings subjected to very varying loads coupled with a moderate amount of vibration and shock, such as would occur in the floors of factories, workshops, warehouses for heavy goods, public halls, etc., an additional allowance equal to one-half the superimposed load. In buildings subject to considerable vibration and shock, such as floors carrying machinery, the roofs of vaults under roadways, etc., an additional allowance equal to the whole superimposed load is made.*

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* See also the L.C.C. Regulations for Reinforced Concrete Buildings.

WEIGHT OF CROWDS.—The importance of the weight of crowds is an element of load which is frequently under-estimated. A very usual and well-established assumption is that dense crowds weigh 100 lbs. per foot super. But recent tests in America point to the necessity of using a higher figure than this. In the tightly-packed lifting cages used in Cornish tin mines, the author has travelled in a party weighing about 137 lbs. per foot super. of cage floor, without serious discomfort, and probably the estimate of Mr Stoney of 147 lbs., or say $1\frac{1}{4}$ cwt., per foot super. is not excessive.

The weight of crowds is a very serious item. Even a large passenger steamer will take a decided list if all the passengers crowd to one side. In road bridges of ordinary width crowds of only 100 lbs. per foot super. will impose a greater load than the largest type of steam traction engine, together with the heaviest load it can draw. A crowd marching in step with a band may also impart impulses which, if synchronising with the natural "period of vibration" of a structure, would tend to set up very violent and dangerous vibrations.

SUPERIMPOSED LOAD OR SUPER LOAD.—The load on any beam or structure is generally considered as being that which is placed upon it, or, as it is sometimes called, the super load; but it is most essential that the designer should never think of it as such without mentally adding the very important item of the weight of the beam or structure itself.

IMPORTANCE OF WEIGHT OF STRUCTURE AS AN ELEMENT OF THE TOTAL LOAD.—Beams whose spans are long as compared with their depth are often using up the greater part of their carrying capacity in supporting their own weight. In the design of the

Forth Bridge, for instance, it was found that by the time the main members had been made strong enough to carry their own weight and the wind pressure, the train loads—the sole object of the bridge's existence—were so small in comparison that there was practically no need to take them into consideration. The weight of all structures, large or small, is a matter which should never be forgotten. Very frequently—in fact in the majority of ordinary cases—it will be found to be negligible as compared with the amount of the superimposed load, and it is for this very reason that it should always be dreaded as a possible danger point. It is an element of the load from which the structure is never relieved, and it has caused more failures, more unsightly saggings and cracks, and probably more fatal accidents than all other causes put together.

Even in a great structure like the St Lawrence Bridge at Quebec, in spite of all the countless precautions, minute care, and constant supervision inseparable from such a gigantic undertaking, it would appear from the report of the Committee appointed to inquire into the reason of the disastrous failure which occurred just before completion, that the primary cause was the dead weight of a comparatively unimportant member. It should never be forgotten that every ounce of material, whether necessary or unnecessary, has to be carried, and therefore reduces the carrying capacity with respect to superimposed loads. It is quite an easy matter to go on adding material with the intention of obtaining a great margin of strength until the structure breaks down simply by its own excessive weight. The object of structural mechanics is to arrange material so that it can do its work under the most advantageous conditions; and the safest way in which material can act is generally the most economical. A structure need not necessarily be massive and unwieldy in order to be strong. It is most strong when its material is in the position best suited to do its work, and under these conditions it is not unfrequently quite light as well as being at the same time quite safe. There is a considerable amount of truth in the American definition of an engineer:—"Someone who can do properly for one dollar what any fool can do for two." The wholesale addition of material to cover up doubtful or neglected stresses is not merely wastefully uneconomical, it is only too liable to be dangerously unsafe.

One frequently notices, for instance, how readily designers will specify a very largely increased depth of concrete when in doubt as to the reliability of ground carrying foundations. Bad ground sometimes, but not invariably, improves at somewhat greater depths; but on the other hand every extra foot in depth of concrete means an additional load of 1 cwt. per foot super. on the ground, and reduces the carrying capacity of the ground by that amount, to say nothing of the very serious additional expense involved, which may amount to as much as 5 per cent. on the cost of the whole building for every extra foot in depth of concrete. In nearly all cases of foundations at ordinary depths a very much stronger, lighter, as well as decidedly cheaper foundation could be obtained by spreading the concrete laterally over a greater area of ground.

FLOOR LOADS.—Except in the cases of warehouses and buildings containing heavy machinery, it is customary in designing columns and piers to accept the extreme improbability of every floor in a building being loaded at the same time to its utmost extent.

Each floor is of course individually calculated for its full load, but from the total weight which a series of floors would transmit to the supporting walls, piers, and columns, a deduction of 10 per cent. is usually made from the total superimposed load of the floor next below the top, 20 per cent. from that of the next floor below, 30 per cent. from that of the next, and so on until the reduction amounts to 50 per cent., above which it is not increased, but is continued at 50 per cent. for all lower floors.

BEHAVIOUR OF MATERIALS UNDER STRESS.—Before considering what margin of safety should be adopted in structures, it is necessary to note the very interesting and very human behaviour of materials under stress.

If a material be given a moderate load, it first gives way slightly but then seems to brace and stiffen up its muscles and carry out its work. Take off the load and it regains its former shape. The work which it was called upon to do having been well within the limits of its elasticity, it performed it cheerfully and without injury. But if material be overloaded, it loses its elasticity and does not recover its shape ; it is injured and has acquired a permanent distortion, or what is called a "permanent set," and is never quite so strong again. It cannot again stand even its usual working load, under which it rapidly becomes more and more decrepit and soon fails altogether.

PERMANENT SET.—This may be illustrated in the behaviour of a piece of ordinary elastic. Loaded with a moderate weight it will stretch, but on the removal of the load the stretch or "strain" will gradually disappear and the original length will be resumed. If, however, the load be increased beyond a certain point it will be found that part of the stretch has become permanent, and upon subsequent applications of even a moderate load this permanent strain or "permanent set" increases in amount, and the elastic will break much more easily than before.

FATIGUE OF MATERIALS.—All materials which are frequently thus overworked seem to suffer from a species of physical breakdown, ultimately failing under loads very much less than would be necessary to destroy healthy normal specimens of the same material. The phenomenon of permanent set and consequent "fatigue" under excessive stress is common to all materials, and it should, if possible, be carefully studied by the student by means of experiments on models of wood and metal beams, wires, and specimens of steel and other materials tested to destruction.

ELASTIC LIMIT AND ULTIMATE STRESS.—The stress at which permanent set takes place is called the "elastic limit," and in most materials it will be found to be somewhere in the neighbourhood of one-half the stress which ultimately causes complete failure, the latter being termed the "ultimate stress."

Obviously, therefore, if material be subjected to a working stress equal to only one-half the ultimate stress, there is no margin of safety left; for under such loading it would—although a long way off complete failure—be on the point of experiencing permanent decrepitude due to overwork.

FACTOR OF SAFETY.—The factor by which the ultimate breaking stress must be divided in order to obtain a reasonably safe working stress is termed the Factor of Safety. It is not a figure which can be readily or carelessly settled, and should always receive careful consideration. If it be fixed too high it involves expensive excess of material, the dead weight of which may well prove dangerous; but on the other hand it must be sufficient to afford *reasonable* security against those inconsistencies in the quality of materials which cannot be estimated, against accidental damage in manufacture, in transit, and in erection, against deterioration from occasional neglect during a normal lifetime, and against temporary overloading. The factors of safety usually adopted have quite enough to attend to in looking after these inevitable and legitimate risks; they have not, and should never be assumed to have, any margin to spare which is available to cover ignorance or laziness on the part of the designer. A factor of safety of 4 (*i.e.*, a working stress of one-fourth the ultimate breaking stress) does *not* leave 75 per cent. of strength doing nothing. As we have seen, out of the 75 per cent. 50 per cent. is mortgaged to the necessity of keeping within the elastic limit, and the remaining 25 per cent. is none too much for what it has to do.

When the student has learned all he can about the forces which attack structures and the forces which resist them, and when he has grasped thoroughly the principles upon which those forces act, he is apt to become somewhat contemptuous about factors of safety and to consider them rather as factors of ignorance. But they are not;—they are the evolution of centuries of failures, and should rather be looked upon with respect as invaluable factors of experience. When we have applied to any problem the whole sum of human knowledge, there still remains the fact that we are bound to assume certain things which may not turn out precisely as we have assumed. Girders are necessarily designed to do their work the right way up, but they do not invariably travel from the yard to the work in that position; occasionally they travel on their sides on a couple of railway trucks, and start their useful career as weight carriers slightly bent by their own weight. Arches and roofs are designed to act under their full distributed load, but under the temporary unequal loading which takes place during erection they are not at all unlikely to be injured. In fact, the sum of minor accidents which can combine to upset the most accurate calculations, the most careful selection of material, and the most excellent workmanship is considerable, and requires a margin of safety which is very different to the clumsy addition of material to cover inaccurate or incomplete calculations.

Factors of safety must necessarily be varied according to circumstances and to the reliance which can be placed upon materials, workmanship, supervision, and the subsequent care and use of a structure When honest workmanship, good materials undamaged in transit, and complete supervision can be absolutely relied upon, the factors of safety in common use might be, and sometimes are, reduced ; but one can very seldom be quite sure, and hidden flaws will sometimes pass the most rigid inspection. If such flaws caused an accident in a case where the usual factors of safety had been reduced, the too confident designer would naturally be held responsible.

The following minimum factors of safety are recommended by eminent engineers for *dead* loads, and they must of course be increased if live loads or shocks are to be expected. They also represent the best materials and workmanship. Ordinary constructional steelwork as used in buildings should have a factor of at least 4. A factor of 3 is of course very low, for a factor of 2 only just keeps on the edge of the elastic limit.

Cast iron, generally	4 (Unwin).
,, pillars	6 (Stoney).
Wrought iron, generally	3 (Unwin and Rankine).
,, roofs	4 (Unwin).
Steel	3 (Unwin and Rankine).
,, ordinary building practice	4 ... ...
Timber	7 (Unwin).
,,	8 (Stoney).
,, exposed to weather	10 (Stoney).
,, for temporary purposes	4 (Stoney).
Brickwork and masonry, generally	20 (Unwin).
Brickwork and masonry, generally	4 to 10 (Rankine).
Arches	20 (Stoney).

It is worthy of note that the earlier engineers advocated less margins of strength in brickwork and masonry than are recommended by engineers of to-day, who have far more extended and more accurate stores of experience to draw from. Rankine—one of the most brilliant of the earlier engineers—gave factors of from 4 to 10 for these materials; whilst half a century later, with bricks and cement of infinitely more reliable character, we find the margins actually doubled by engineers of the first rank.

Designers should, therefore, beware of being tempted as their knowledge grows to cut down the ordinary margins of safety. Whilst it is not only dishonourable but dangerous to attempt to provide (with material which one's client pays for) against ignorance of what a structure has to do and how it is going to do it; yet on the other hand, when all possible stresses have been calculated and all proper precautions taken, the margin dictated by experience will *still* be required; and the final result often has a slender boldness which appears to be almost uncomfortably daring.

CHAPTER IV

LEVERAGE AND TURNING MOMENTS

Simplicity of True Structural Design—Method of Solution of Structural Problems by Simple Equations—Leverage—Paradox of Leverage—Work—Mechanical Advantage—Relation between Mechanical Advantage and Leverage—Examples of Simple Leverage and Calculation of Turning Moments—Importance of Setting Down Equations in Identical Terms.

SIMPLICITY OF TRUE STRUCTURAL DESIGN.—There is a very striking difference between the extreme simplicity of the practical calculations which are used by men thoroughly well versed in the science of structural design, and accustomed to its practice and responsibilities, and the extreme complexity of those which are advocated by many text-books and taught in many lecture-rooms. The structural engineer in busy practice uses his slide rule constantly, for he calculates much and leaves as little as possible to judgment ; but the integral calculus he uses only when he must—an extremely infrequent contingency. This is not because constant practice has made him callous as to his responsibilities, for rule-of-thumb finds no favour with such men. It is that because, knowing their subject thoroughly, they therefore know its real simplicity and do not apply higher mathematics to problems which can be solved with equal accuracy, less chance of error, and far greater ease and lucidity by means of simple arithmetic.

When one follows the really beautiful if complex analyses and deductions of many mathematical writers, it is hard to realise that the principles of leverage expressed in simple equations will enable us to arrive rapidly at what are, for the purposes of practical design, precisely the same results.

Mathematical analysis is a splendid instrument, by means of which wonderful results of practical and far-reaching utility have been attained. In research work it is invaluable. But to use higher mathematics for the solution of simple problems in practical design is like using a razor to chop firewood; one gets far better and quicker results with an axe. It will be found that there is hardly any ordinary practical problem in structural design which cannot be solved by means of simple equations, provided that the designer is acquainted with the principles of leverage, with the behaviour of materials under stress, and with a few similar physical facts which can be easily learned from observation.

METHOD OF SOLUTION OF STRUCTURAL PROBLEMS BY SIMPLE EQUATIONS.—The method by which simple equations are utilised needs very little explanation. When it is known that one group of factors has the same net total value as another group, then if one of the groups contains a factor whose value is unknown, that value can be readily deduced if the two groups be arranged in the form of an equation.

Thus, if it be known that $A \times B = C \times D$, and the values of A, C, and D only are known, then B can be quickly computed, for $B = \frac{C \times D}{A}$. Whenever, therefore, it is desired to ascertain any unknown stress or reaction, the known stresses and reactions are always arranged in the form of some convenient equation; and then a very little simple arithmetic or a few movements on a slide rule quickly gives the desired information. But stress and reactions can only be readily arranged in their true relationship when the principles of leverage have been thoroughly grasped, and the student will be well advised to spend some little time in carefully investigating those principles, even though he may believe himself to be well acquainted with them, and may feel that he is being dragged laboriously along a well-known path. Considerations of leverage enter so constantly and bulk so largely in structural problems that the acquisition of a habit of mind which instinctively and correctly applies them to every problem is well worth the expenditure of some little time and trouble. Both are quickly repaid when one comes to actual design.

PARADOX OF LEVERAGE.—On commencing to investigate the principles of leverage one is met by a curious paradox. Two children on a see-saw, one very light and the other heavy, will balance if the lighter child sits further away from the centre than the heavier child does. Why this should be so is not apparent, there being no additional force like friction to help the lighter weight or to hamper the heavier one.

In the same way that reaction is easy to comprehend if considered in connection with force exercised by a moving body, as in the case of a kick given to a brick wall, so it will be found that the reason of the apparent paradox of leverage is clear if studied in connection with the mechanical advantage which can be given to moving bodies for the purpose of doing work.

WORK.—Work may be defined as power or force exercised through a given distance, and is measured in foot-lbs. without reference to the time taken over the operation. The work involved in raising a 2-lb. weight 10 ft. high, 20 ft.-lbs., in a minute, is the same as that which is involved in raising a 4-lb. weight 5 ft. high in half a minute. The accomplished work is the same although the commercial value of the operation differs.

A weak man can do the same work as a man of twice his strength, if he can do it in a way that will enable him to take his time over it and so exercise his weaker muscular power through *double the distance.* He will, of course, in consequence take twice as long over it, and his time will therefore be worth per hour only half that of the strong man. *But he will be able to get the task done.* Suppose, for instance, that it consist of carrying from the bottom to the top of a hill a load of 1 ton divided into eighty packages of 28 lbs. each. The strong man could take two packages at a time, and need therefore only make forty journeys. The weaker man, able only to take one at a time, would require to make eighty journeys. But *because* he is thus able to exert his weaker muscular force through a greater distance, he is able to achieve the ultimate object of transporting 1 ton from the bottom to the top of the hill. Had the load been in 56-lb. packages he would have found the task impossible.

MECHANICAL ADVANTAGE.—Imagine a couple of men on the deck of a ship hauling up a spar and sail weighing, say, half a ton, or 1,120 lbs. The men weigh, say, 150 lbs., or about 11 stones, each. They could not possibly exert a downward pull of more than this without lifting themselves up off the deck, and yet with a suitable arrangement of blocks and falls they can and do lift the half-ton load easily. The reason is that the arrangement of the lifting rope through the blocks enables them to haul in, say, 2 ft. of rope for every 4 in. that the spar lifts—a "mechanical advantage" of six to one. 1,120 lbs. lifted through 4 in. only represents the same amount of work, 4,480 inch-lbs., as $186\frac{2}{3}$ lbs. lifted through 24 in. So that each man need only exert a pull of, say, 14 lbs., in order to overcome the friction of the blocks, ropes, etc., and move the half-ton weight. Because they can exercise their force through a greater distance, the task is within their powers. There is no new force introduced by the blocks, they merely enable the men to exercise their muscular power over a greater distance. The work put on to the hauling rope is the same as that which is given out by the lifting rope, except what is lost in friction. Now note that if the men pull briskly the spar moves quickly; if they want to move slowly the spar moves slowly. They can work more and more slowly until at last they can stop and balance the half-ton spar by a pull of about 185 lbs. on the hauling rope (the friction which always opposes movement now helping them). The obvious mechanical advantage which they had when moving does not disappear when they slowed down to an absolute stoppage. Then the conditions were the same as those of the see-saw, a smaller weight balancing a larger, because the smaller is given a mechanical advantage or leverage.

RELATION BETWEEN MECHANICAL ADVANTAGE AND LEVERAGE.—Mechanical advantage and leverage are identical whether in moving or in still bodies. Conditions governing the tendency to move are the same as those governing actual movement.

The same thing can be studied in the two pulleys shown in Fig. 7 which are arranged so that the lighter

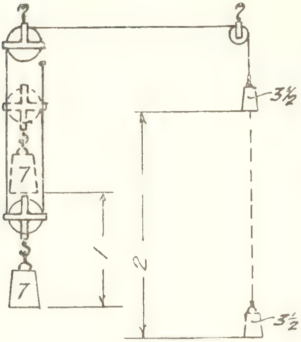


Fig. 7.

weight travels vertically downwards through double the distance that the heavier weight travels upwards. Under these conditions if the pulleys be well oiled and run easily, a weight and pulley of 7 lbs. will be pulled up by a weight of very slightly over $3\frac{1}{2}$ lbs., and will be balanced by a weight slightly under $3\frac{1}{2}$ lbs.—the difference being caused by friction as in the last example.

It can also be illustrated in the action of a differential pulley block by which a labourer will lift a huge block of stone (which if unaided he could not budge) by hauling in several feet of chain for every inch that the stone moves upwards.

Take now the case of a 12-in. grooved wheel on a 2-in. spindle, as shown in Fig. 8. Round the 12-in. wheel is fastened a cord carrying a 1-lb. weight, and round the 2-in. spindle a cord carrying a 6-lb. weight. The circumferences of the two circles being, like their diameters, in the proportion of six to one, if the heavier weight could pull the spindle round in spite of the smaller weight, it would move downwards through one-sixth the distance that the lighter weight moved upwards. If the wheel weight overbalanced the spindle weight, the former would similarly move downwards through a distance six times as long as that through which the spindle weight moves upwards. If the whole be accurately mounted on centres it will be found that a very slight addition to the smaller weight will suffice to lift the larger weight. When they balance and are still, the mechanical advantage remains the same. *The respective mechanical advantages or leverages of all forces are measured by the lengths of the paths along which they travel or tend to travel:* and in the case of forces tending, as in this case, to rotate a body or structure about a given point, the lengths of these paths are the circumferences of the respective circles of rotation. The circumferences being in turn proportional to the radii, it will be sufficient to consider only the radii; which, as will be easily seen, form a simple lever with arms in the proportion of six to one; and may be taken as a scale model of the see-saw with the lighter child sitting well away from the centre balancing the heavier child sitting nearer to it.

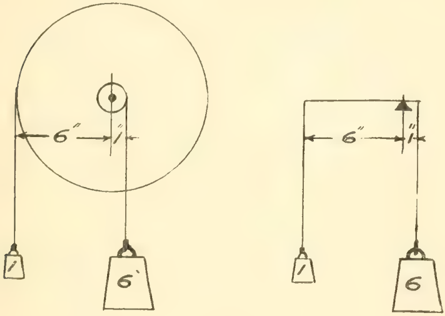


Fig. 8.

Examples of Simple Leverage and Calculation of Turning Moments.—Every student who has not the advantage of access to a physical laboratory where the accuracy of calculated reactions, bending moments, etc., can be proved, should at least provide himself with a long balanced lever or even a 5-ft. rod which he can hang in a string loop, a few weights, and, if possible, a spring balance, and take the trouble to prove as many as possible of the following considerations of leverage. He will not find the time wasted.

If a 7-lb. weight be hung 12 in. from the fulcrum of the simple lever shown in Fig. 9, in which the effect of friction is negligible, and a spring balance be applied 12 in. from the fulcrum on the other

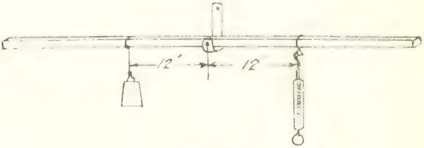


FIG. 9.

side, it will be found that a pull of 7 lbs. will effect a balance. If the weight be hung 2 ft. from the fulcrum a pull of 14 lbs. is required. If the distance be halved, $3\frac{1}{2}$ lbs. suffices for a balance. In each case the spring balance pull and the weights are tending to move in circular paths round the pivot. Obviously, it is not the weights or pulls which balance, except when they are acting at the same distance from the fulcrum, *i.e.*, when they are tending to travel on paths of the same length. When the lengths of their respective paths vary, the advantage or disadvantage of the variation must be taken into account. This is done by multiplying the force by the relative length of its path, the result, called the "moment" of the force, representing its true relative efficiency. But for purposes of comparison (which are the only purposes for which moments are used) it will be more convenient and equally accurate to multiply each force by the *radius* of its circular path of action as by the *actual length* of that path, the radius being always a definite proportion of the circumference.

When comparing forces tending for and against the rotation of a body or structure about any given point it is not only necessary to compare the *amount* of the forces—their respective mechanical advantages must also be taken into consideration. The turning *moments* in one direction must be compared with the turning *moments* in the contrary direction.

In the simple experiment just described, the turning moments of the weight tending to turn the lever in an anti-clockwise direction at the distances of (*a*) 12 in., (*b*) 24 in., and (*c*) 6 in. from the fulcrum would be (*a*) 84, (*b*) 168, and (*c*) 42 in.-lbs. respectively. The contrary moments in a clockwise direction necessary to balance these must be the same.

Anti-clockwise Moments ↺ = Clockwise Moments. ↻
(*a*) 84 in.-lbs. (7 lbs. × 12 in.) = 7 lbs. × 12 in. = 84.
(*b*) 168 ,, (7 ,, × 24 ,,) = 14 ,, × 12 ,, = 168.
(*c*) 42 ,, (7 ,, × 6 ,,) = $3\frac{1}{2}$,, × 12 ,, = 42.

Importance of Setting Down Equations in Identical Terms.—Whenever a calculation of moments has been made it should invariably be reviewed to see that all the factors have been calculated in the same terms—that some dimensions in inches have not been multiplied by others in feet, that cwts. on one side are not compared with tons on the other side, and so on. This is a slip which is very easily made, and is a most frequent source of error, even in the work of experienced designers; and the habit of revising even the most simple calculations is well worth acquiring.

If now several weights be hung on each arm of the lever it will be found that they will only balance when they are in such positions that the total of the clockwise turning moments equals the total of the anti-clockwise moments.

The following is a typical example of the use of equations in solving structural problems.

On the right-hand side of the fulcrum hang three unequal weights at unequal distances, say 1-lb., 4-lb., and 2-lb. weights at 12, 18, and 20 in. respectively from the fulcrum; and on the left-hand side hang one 4-lb. weight at 15 in. from the fulcrum. At what distance would a 2-lb. weight require to be hung to effect a balance?

↻ CLOCKWISE MOMENTS = ANTI-CLOCKWISE MOMENTS. ↺

$$(12\text{in.} \times 1) + (18\text{in.} \times 4) + (20\text{in.} \times 2) = (4 \times 15\text{in.}) + (2 \times X\text{in.})$$

$$= 12 + 72 + 40 = 124 \text{ in.-lbs.} = 60 + 2X \text{ in.}$$

$$X = \frac{124 - 60}{2} = 32 \text{ in.}$$

Under the same conditions what weight would be required to effect a balance if hung at a distance of 8 in. from the fulcrum?

$$124 \text{ in.-lbs.} = 60 + 8X \text{ lbs.}$$

$$X = \frac{124 - 60}{8} = 8 \text{ lbs.}$$

The student will, without difficulty, be able to devise a number of similar problems and to prove his results by actual experiments.

CHAPTER V

LEVERAGE AND TURNING MOMENTS (*continued*)

Calculations of the Reactions at the Bearings of Irregularly Loaded Beams—Rule for same—Method of Calculating Turning Moments—Importance of Direction of a Force on a Body—Point of Application Immaterial—Centre of Gravity—Method of Location of Centre of Gravity by Plumb Bob Lines—Calculated Location of Centre of Gravity by Method of Moments.

CALCULATIONS OF THE REACTIONS AT THE BEARINGS OF IRREGULARLY LOADED BEAMS.—The calculations of the reactions at the bearings of irregularly loaded beams should now be a simple matter. Let one-half of the 74-in. jointed rod shown in Fig. 10 be laid across two knife-edge bearings—the other half hanging free, but firmly screwed up at A. Let the portion between the bearings be loaded irregularly with, say, 7 lbs., $8\frac{1}{2}$ lbs., and 5 lbs., at 10, 20, and 24 in. respectively from A.

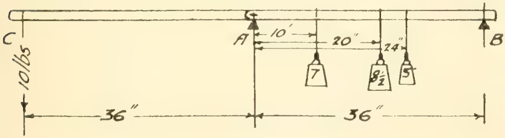


FIG. 10.

What pull on the free end C, 36 in. from the bearing, will just lift the rod off the right-hand bearing at B?

The bearing A will obviously be the centre of rotation about which turning moments must be calculated, the weights between A and B tending to cause a clockwise rotation about A, and the unknown pull X an anti-clockwise rotation about the same point.

$$\curvearrowright \text{ Clockwise Moments } = \text{ Anti-clockwise Moments } \curvearrowleft$$

$$(7 \times 10 \text{ in.}) + (8\tfrac{1}{2} \times 20 \text{ in.}) + (5 \times 24 \text{ in.}) = X \times 36 \text{ in.}$$

$$X = \frac{70 + 170 + 120}{36} = 10 \text{ lbs.}$$

If the rod be so light that its weight may be neglected, then the fact of the portion A B being in line with the portion A C does not affect the calculation at all. Let the two halves be fixed at right angles by the thumbscrew joint over bearing A, as in Fig. 11, then *so long as the pull at C be at right*

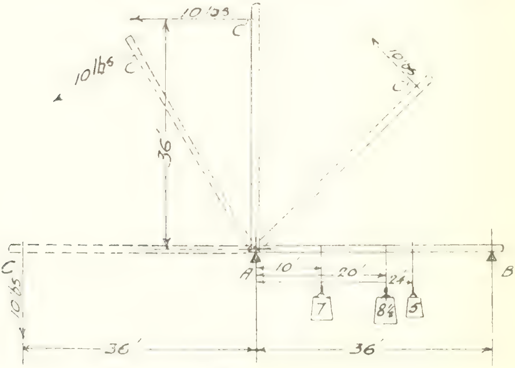


FIG. 11.

angles to the rod, it will be found to be as before, 10 lbs. This will be the case at whatever angle the rod be clamped. It may even be bent double as in Fig. 12, so that C and B are identical: a 10-lb. pull still just lifts the loaded beam at B. In other words, it just equals the reaction at B and is able to take its place.

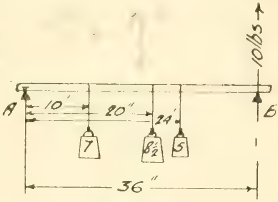


FIG. 12.

If one bearing of a loaded beam were suddenly removed, the first impulse of the beam and loads would be towards rotating round the other bearing as a pivot. This tendency, therefore, is what the reaction at the first bearing has always to resist, and the leverage at which it can act for that purpose is the radius of the circle of rotation, *i.e.*, the span. So that the reaction at B may be considered as an anti-clockwise force acting at a leverage A B against the cumulative tendency of the weights between A and B to rotate round A as a centre. The reaction owes its existence solely to the tendency of the beam and loads to rotate about A, and must, when multiplied by its mechanical advantage, be equal to the *moments* of the forces which call it into being, or,

$$\text{Reaction at B} \times \text{span} = \text{sum of clockwise turning moments of beam and loads about A.}$$

Or,

$$\text{Reaction B} = \frac{\text{Sum of clockwise turning moments of beam and loads about A}}{\text{span}}$$

In the same way the reaction A = the sum of anti-clockwise turning moments of beam and loads about B divided by the span.

RULE FOR CALCULATING REACTIONS FOR IRREGULARLY LOADED BEAMS. — To ascertain, therefore, the reaction at either bearing of an irregularly loaded beam, treat it as a force tending to rotate the beam and loads around the other bearing, and opposing the tendency of the weight of beam and loads to rotate in a contrary direction. In other words, multiply each element of the load by its distance from the *other* bearing, add the results together and divide by the span.

It is necessary to carefully keep in mind that all distances must be measured from the assumed centre of rotation. It is easy to make a serious error by measuring the leverage of a load from the wrong bearing. It will be obvious that the two reactions together equal the total weight of beam and loads ; and that if one reaction be calculated, the other can be ascertained by merely deducting the first calculated reaction from the total load. This somewhat slack method of calculation is often responsible for mistakes which are by no means easy to detect. Both reactions should invariably be calculated separately, and the total load should then, and not till then, be ascertained as a check on the results. The student should arrange and calculate a number of examples of

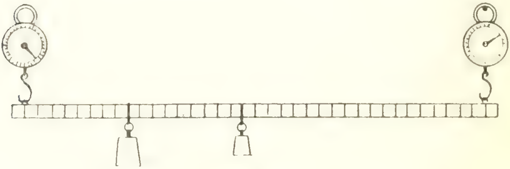


FIG. 13.

irregular loading at various spans on a rod as shown in Fig. 13, checking the results against the readings of the spring balances. The circular balances shown indicate the reactions rapidly and accurately, but the same results can be obtained by placing the loaded rod on solid bearings, and ascertaining the end reactions by means of a spring balance and a string loop.

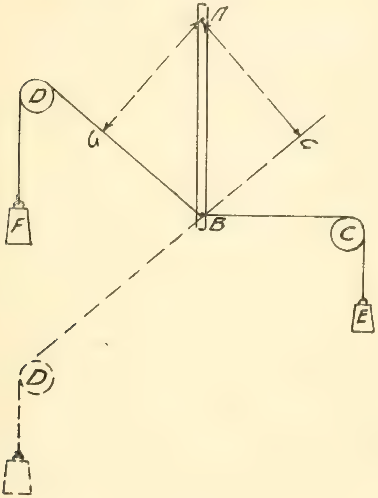


FIG. 14.

METHOD OF CALCULATING TURNING MOMENTS.—Let a strip of wood or cardboard A B (Fig. 14) be hung so that it can rotate freely on a pin at A. At B let two forces be applied by weights E and F fastened to silk cords passing over well-oiled and easily running pulleys at C and D. Let the pull B C be at right angles to A B, and the pull D B be at any other angle.

The weight E tends to rotate the strip round A in an anti-clockwise direction, and its turning moment is obviously E × A B, the radius of the circle of rotation which E tends to cause. It will be found that the weight at F required to balance this turning moment is always greater than E whenever the pulley D is placed in such a position that the angle D B A is greater or less than a right angle. Obviously the pull D B under those circumstances must be working at a disadvantage as compared with the pull B C. Therefore its mechanical advantage or lever arm must be shorter. It will be found that the pulls balance when, and only when, the pull in B C × the distance B A between it and the centre of rotation A *measured at right angles to the direction B A* = the pull in D B × the distance G A between it and the centre of rotation A *measured at right angles to the direction D B.* The lever arm of a force, *i.e.*, the radius of the circle of rotation which it tends to cause, must *always* be measured *at right angles to the direction of the* force. This is only what would be expected, because if a force exercises a turning moment, the circle of rotation which it tends to cause would be tangential to the line of direction of the force—and as a tangent to a circle is necessarily at right angles to the radius at the point of contact, a line at right angles to that direction of the force and passing through the centre of the circle would be a radius of the circle of rotation. It is, however, just as well that this simple but important point should be proved and firmly fixed in the mind by means of practical experiment: in everyday work its very simplicity seems to constantly invite draughtsmen to make unaccountable mistakes over it.

IMPORTANCE OF DIRECTION OF FORCE ON A BODY.—Let an irregular piece of cardboard be pinned to a board securely but free to rotate easily, let forces be applied as shown in Fig. 15 tending to cause rotation in contrary directions, and let those forces be adjusted until the cardboard comes to rest under their combined moments. If the direction of the forces and the corresponding lever arms be traced on the cardboard as shown in dotted lines, and each weight be multiplied by the length of its lever arm to obtain its turning moment, it will be found that the sum of the clockwise turning moments exactly equals the sum of the anti-clockwise moments (the essential condition of equilibrium) only when the lever arms are measured *at right angles to the direction of the forces*. This should be varied several times by slightly altering any one of the weights, an operation which causes all the other

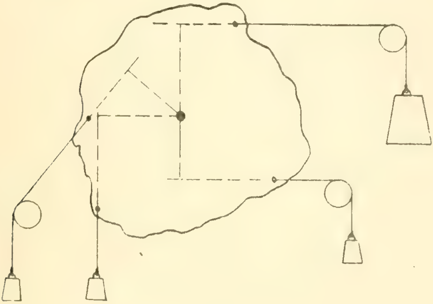


FIG. 15.

weights to take up new lines of direction with corresponding new lever arms.

POINT OF APPLICATION IMMATERIAL.—It will be seen that the centre of rotation and the *direction* and *amount* of each force are the only important factors—the particular point at which the force may be applied to the fabric of the body or structure is of no importance whatever, and can be altered indefinitely so long as the alteration does not affect the *direction* of the force, or the length of the lever arm. Let a number of points along the line of direction of any one of the forces when balanced be marked on the cardboard, and it will be found that the cord can be fastened to any one of them without altering the state of equilibrium — whereas the slightest alteration in the weights in any scale pan or in the direction of any force (as by deflecting a cord by means of a pin) would upset it at once.

CENTRE OF GRAVITY.—The centre of gravity of a body is that point about which a body will balance in any position. It is the point about which all the different turning moments exercised by the weight of every atom of the structure are equal. In structural mechanics we are generally concerned only with the rotation of bodies of equal thickness in a plane at right angles to that thickness, and we require only to locate the position of the centre of gravity with reference to one view, generally a cross section of the body. The centres of gravity of regular geometrical figures, such as squares, triangles, parallelograms, are known. In circles the centre of gravity is at the centre, in a parallelogram it is at the intersection of the diagonals, in a triangle at the intersection of any two lines joining an angle with the centre of the opposite side, and is always at one-third the altitude measured from any side. If such figures be cut out in cardboard and loosely pinned to a board at the points mentioned, they will balance readily in any position, whereas if pinned at other points they will not.

METHOD OF LOCATION OF CENTRE OF GRAVITY BY PLUMB BOB LINES.—The centre of gravity of irregular forms may be obtained by cutting out the form in cardboard, hanging it loosely on a pin, and tracing from the pin a vertical line by means of a plumb bob and fine cord, as Fig. 16. It will be found that wherever the pin be placed all such vertical lines intersect at the same point, which will be found on trial to be the centre of gravity. The reason for this can readily be seen by considering the conditions of balance when the figure is hung from any point. In this position all the turning moments exercised by the various atoms of weight to the right of a vertical line from the pin must be exactly balancing all those on the left of it. But this position is only one of the innumerable positions in which the figure will balance about the centre of gravity when that has been found: and when it does eventually so balance about the centre of gravity, the sum of the moments

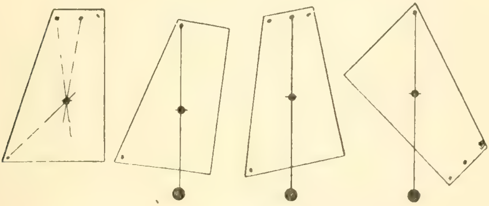


FIG. 16.

on the right of the centre of gravity will exactly equal the sum of the moments on the left of it. This is precisely what they are doing in the position under consideration, and as the vertical weights are the same in both cases, and the lever arms, measured at right angles to the directions of those weights—viz., horizontally—are of the same length, it necessarily follows that the centre of gravity wherever it is must be somewhere along the vertical plumb bob line. For if it were anywhere on either side of the plumb bob line the lever arms would not be the same length in both cases. If now a fresh position of the pin and a fresh position of the plumb line be taken it is equally true for similar reasons that the centre of gravity must also be along the second plumb bob line, and the only position where it can possibly be on both lines is at their intersection. It will probably have already occurred to the student that a distributed weight exercises the same *turning moment* as a concentrated weight of the same amount would do if it acted at the centre of gravity of the distributed weight. This important

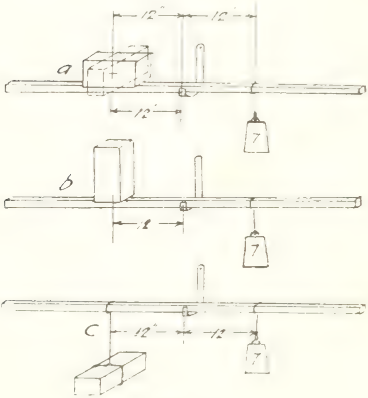


FIG. 17.

point can readily be proved as follows. Let an ordinary brick be weighed and the centre lines of all its sides marked. If it weigh, say, 7 lbs. and be placed on a lever as shown in Fig. 17, with the centre of the 9-in. side 12 in. from the fulcrum, it will be found that a weight of 7 lbs. at 12 in. from the fulcrum on the other arm, or $3\frac{1}{2}$ lbs. at 24 in., will be required to balance. It exercises, therefore, a turning moment of 84 in.-lbs. Now let it be turned round so that it rests on its $4\frac{1}{2}$-in. side, Fig. 17 (*a*), the moment will be the same and the balance will be unaffected. The same obtains if it be stood up on its 3-in. side, Fig. 17 (*b*), and also if it finally be hung, as in Fig. 17 (*c*), in a string loop from the 12-in. mark.

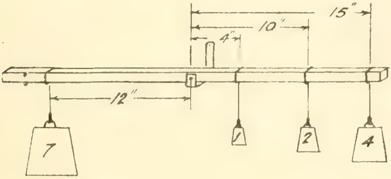


FIG. 18.

CALCULATED LOCATION OF CENTRE OF GRAVITY BY METHOD OF MOMENTS.—The centre of gravity of a ring is obviously in the air, as would also be the common centre of gravity of two widely separated bodies. In fact the centre of gravity of any figure is merely the common centre of gravity of all its separate particles. The centre of gravity of several parallel forces can be similarly considered. Let several unequal weights, say 4 lbs., 2 lbs., and 1 lb., be hung on one arm of a lever as in Fig. 18 at different distances, say 15 in., 10 in., and 4 in. respectively, from the fulcrum, and a weight equal to the total of those weights be slid along the other arm until a balance be effected. This would be at—

$$\frac{(4\text{ lbs.}\times 15\text{ in.})+(2\text{ lbs.}\times 10\text{ in.})+(1\text{ lb.}\times 4\text{ in.})}{7\text{ lbs.}}=12\text{ in.}$$

from the fulcrum. The 7 lbs. concentrated weight on one side of the fulcrum balances the 7 lbs. of scattered weights on the other side. As a distributed weight exercises the same moment as a concentrated weight of the same amount acting at the centre of gravity of the distributed weight, it follows that the centre of gravity of the scattered weights must be 12 in. from the fulcrum. If a point 12 in. to the right of the fulcrum be marked, the concentrated weight removed, and the several weights rearranged at the same relative distances from the fulcrum as they were from the 12-in. mark, as in Fig. 19, they will be found to balance, and the turning moments of each side if calculated will be found to be equal. If, therefore, it be desired to find the centre of gravity of any number of parallel forces they should be imagined as balanced by a weight equal to their sum over a fulcrum anywhere on a weightless lever rod. Then the moment of each force about that fulcrum is found by multiplying the amount of each

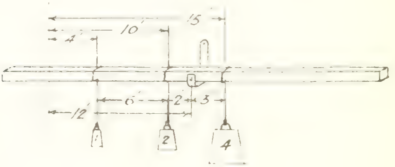


FIG. 19.

force by its distance from the fulcrum. The sum of all these moments are balanced by a concentrated force *of the same total amount*, which must, therefore, act at a leverage equal to the distance from the fulcrum to the centre of gravity of the separate parallel forces. If, therefore, the sum of the moments be divided by the sum of the weights, the result is the distance of their centre of gravity from the fulcrum about which the moments were calculated, and the required position of the centre of gravity is thus defined.

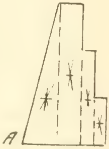


FIG. 20.

This method of finding centres of gravity is very useful, and its accuracy should be proved by a number of experiments. It can be applied to finding the centre of gravity of figures, and is not only a useful check on the plumb-bob method, but is often much quicker and more accurate than the latter. Its results can also be readily proved by recalculating from a new fulcrum. In the case of the retaining wall with a stepped back and a battered face, shown in Fig. 20. This can be divided into the four parts shown, whose area and whose individual centres of gravity can be easily ascertained. The weights of these figures being proportional to their respective areas, the horizontal distance of their common centre of gravity from any point, say the outside toe, A, can be calculated, and a vertical line drawn at this horizontal distance from A. For many purposes the location of this line is all that is required, but if it is required to locate the position of the centre of gravity upon this line—then its vertical height from any point, say the base, can be calculated by taking moments about that line as a fulcrum.

CHAPTER VI

ABBREVIATED METHODS OF CALCULATION

LOGARITHMS AND THE SLIDE RULE

Calculation by Logarithms—The same Performed by Mechanical Methods—The Slide Rule: Simplicity, Ease, and Rapidity of Working—Wide Range of Calculations which can be Performed—Extraction of Square and Cube Roots—Sines and Tangents—Metrical and Other Calculations.

THE one really unpleasant part of the study and practice of structural mechanics is the large amount of tedious simple arithmetic which it involves. To the busy man, whether he be student, draughtsman, or chief, calculations are a nuisance; whilst those who dislike figures and have a strong penchant for graphic short cuts are apt to become speedily nauseated by large doses of simple arithmetic. To such the use of a slide rule is a real necessity, and to everyone who has much to do with figures it is a most useful economiser of time and mental effort. At the same time, the degree of accuracy which is possible with its reasonably careful use is quite sufficient for the purposes of ordinary practical design.

CALCULATION BY LOGARITHMS.—The ordinary slide rule is based on the principle of tables of logarithms. By the use of logarithms, multiplication and division can be performed by simple addition and subtraction; and the calculation of squares, cubes, and other powers, and the extraction of square, cube, and other roots can be performed by simple multiplication and division. Logarithms are numbers which are so calculated with regard to ordinary numbers that the logarithm of an ordinary number (a) added to the logarithm of another ordinary number (b) gives the logarithm of a multiplied by b.

Similarly, $\log. a - \log. b = \log. \frac{a}{b}$.

Also, $\log. a \times 6 = \log. a^6$,

and $\log. a \div 3 = \log. \sqrt[3]{a}$,

or, $\log. a \div 3.12 = \log. \sqrt[3.12]{a}$.

Thus to multiply 523.6 by 3.1416, all that is necessary to do is to find in a table of logarithms the logarithms of those two numbers and to add them together; the result will be the logarithm of the product required.

$$\text{Log. } 523.6 + \text{log. } 3.1416 = \text{log. } (523.6 \times 3.1416).$$

$$\begin{aligned} \text{Log. } 523.6 &= 2.7189996 \\ \text{Log. } 3.1416 &= \underline{0.4971509} \\ &\ 3.2161505 = \text{log. of } 1644.942. \end{aligned}$$

To divide 57.2958 by 0.7854.

$$\text{Log. } 57.2958 - \text{log. } 0.7854 = \text{log. } \frac{57.2958}{0.7854}.$$

$$\begin{aligned} \text{Log. } 57.2958 &= 1.7581228 \\ \text{Log. } .7854 &= \underline{\bar{1}.8950909} \\ &\ 1.8630319 = \text{log. of } 72.9511. \end{aligned}$$

To find the cube root of 1696.5.

$$\frac{\text{Log. } 1696.5}{3} = \sqrt[3]{1696.5}.$$

$$\frac{\text{Log. } 1696.5}{3} = \frac{3.229554}{3} = 1.076518 = \text{log. of } 11.927.$$

CALCULATION BY LOGARITHMS PERFORMED BY MECHANICAL METHODS.—These three examples will suffice to show how enormously the labour of arithmetic can be reduced by means of logarithms; but the slide rule goes further. It not only automatically extracts from the tables any given logarithms, but it also performs the operations of adding and subtracting them to any extent mechanically without calling for any mental effort on the part of the operator beyond that of reading a scale; and then it gives the answer, not in a logarithm, but in a natural number.

It dispenses with the use of logarithmic tables by being divided into lengths which correspond to the *logarithms* of different natural numbers; but those lengths are marked with their corresponding *natural* numbers, so that whilst the operator is apparently reading natural numbers on the scale he is really reading lengths corresponding to the logarithms of those numbers.

THE SLIDE RULE—SIMPLICITY, EASE, AND RAPIDITY OF WORKING.—The mechanical operation of addition and subtraction by the slide rule is performed as follows:—

Two scales are used, the edges of which slide past each other. Let *a* and *b* (Fig. 21) be two views of such sliding scales.

The scales being similarly divided and marked, each division and marking on the lower scale when in the position *a* is directly under a similar division and marking of the upper scale. If the lower scale be moved to the right as in *b* until its left-hand end reaches, say, 3 in the upper scale, then every division on the lower scale will be directly under a division on the upper scale, which will be further to the right by the length 1-3. If all the divisions on both scales had been of equal length every figure in the lower scale would have been under a figure on the upper scale greater by 3. Thus 4 on the lower scale would have been under 7, and 6 under 9, and so on. But the divisions being not numbers as marked, but the *logarithms* of those numbers, they vary in length; and the operation is that of adding together the logarithms of the scale

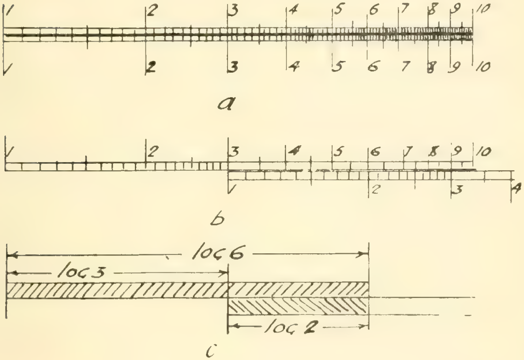


FIG. 21.

reading as in Fig. 21, *c*. The result of adding together the logarithm of 3 and 2 would be to produce the logarithm of 3 × 2 = log. 6. So that when the lower scale is moved forward bodily to the extent of the logarithm of 3 every reading 2, 3, 4, 6, etc., will be under a division of the upper scale, which would be the logarithm of 2, 3, 4, or 6 added to the logarithm of 3. These logarithmic divisions being marked with the corresponding *natural* numbers we find 2 under (2 × 3) or 6, 3 under (3 × 3) or 9, 6 under (6 × 3) or 18, and so on.

Division by the slide rule is the same process reversed.

With the scale in the position shown in Fig. 21, *b*, the logarithm of 6 has had cut off from it a length equal to the logarithm of 2. What remains must therefore be log. 6 − log. 2, *i.e.*, log. $\frac{6}{2}$ = log. 3.

It will be observed that each division on the scale is divided into ten parts. This enables the scale to be read in decimals, but as there is nothing to denote the position of the decimal point, any given reading such as 216 might be 2.16, .00216, or 21,600. It is always necessary when calculating by means of logarithms to fix independently the position of the decimal point in the result. When the calculation is made by means of tables of logarithms it is possible to do this by means of a simple system of indices. But with the logarithmic slide rule this is not practicable. For simple calculations the position of the decimal point is frequently obvious, and in those which are more complicated it can always be rendered so by writing down the calculation in some very simple but similar form.

For instance, for the calculation of—

$\frac{2.66 \times 20 \times 0.3}{8}$ the slide rule simply gives the result as 2.

A simple but similar form would be—

$$\frac{50}{8} \times 0.3 \text{ or } \frac{15}{8}.$$

Obviously the result is not 20 or 0.2 but 2.

The Wide Range of Calculations which can be Performed on a Slide Rule.—The ordinary flat slide rule is generally made in the form shown in Fig. 22, and consists of a strip of wood or celluloid marked with two scales tongued and sliding in double grooved rebates in a wider strip marked with two corresponding scales, the faces of both strips being flush and the divisions of the corresponding scales marked along the touching edges.

The lower scales are similar to the upper, but the divisions are larger, and therefore can be read with a greater degree of accuracy; although being in consequence less numerous, they are not so convenient as the upper scale for solving equations consisting of a large number of terms.

The lower scales have only half the number of divisions as the upper scales, they are of the same total length, and their ends correspond. The result is that 4 in the upper scales would be over 2 in the lower scales, 6 over 3, and so on; and any reading, say 4 on the upper scales, would be vertically over $\frac{\log. 4}{2}$ on the lower. But $\frac{\log. 4}{2} = \log. \sqrt{4}$, so that all the readings on the lower scales are the *square roots* of the readings immediately over them of the upper scales, and the extraction of the square root of any number by means of the slide rule is therefore an extremely simple matter.

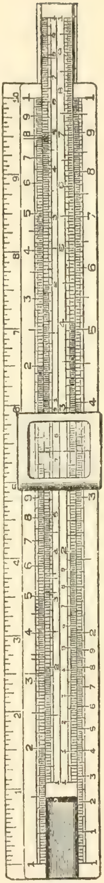


FIG. 22.

The extraction of cube roots, which is slightly less simple but entails very little more trouble, is described at the end of the chapter, whilst the raising of a number to any power other than a decimal power can be performed by the movements which effect simple multiplication.

The great majority of calculations in structural mechanics consists of the solution of simple equations of several terms. Thus—

$$\frac{W \times 9 \times 12}{8} = \frac{3 \times 9 \times 9 \times 9}{6},$$

$$W = \frac{3 \times 9 \times 9 \times 9 \times 8}{6 \times 9 \times 12} = 27.$$

The operation of performing compound calculations of this type is greatly assisted by a transparent index called a cursor and marked with a vertical hair line. It is clipped into grooves formed in the outside edges of the wider strip, and slides over the flush faces of both strips and thus serves to temporarily mark any given reading in the outer scales, whilst the inner strip is being moved under it into the correct position for dividing that reading by any number. For instance, if it were desired to divide 3.64 by 1.03. To set 1.03 under 3.64 would try the eyes decidedly, but the hair line in the transparent cursor can easily be moved until it lies exactly over the reading 3.64 on the outer scale, and then the inner scale can be moved until 1.03 also corresponds with the now stationary hair line. Zero on the sliding scale would then point to $\frac{3.64}{1.03}$ or 3.534.

Extraction of Square and Cube Roots.—As the divisions on the lower scales are double the length of those on the upper it is obvious that the former must represent the logarithms of the squares of the latter. But most slide rules are so divided that there are on the upper scales two readings of any number, whilst the corresponding readings on the lower scale differ. If it were required to ascertain the square root of 81. There are two 8_{10} readings on the upper scale, one on the left-hand half, and the other on the right. Under these readings we find 2_{85} and 9_{00} respectively. Further investigation shows that:—

$\sqrt{.081} = .285$ and $\sqrt{.81} = 0.9$

$\sqrt{8.1} = 2.85$,, $\sqrt{81} = 9$

$\sqrt{810} = 28.5$,, $\sqrt{8100} = 90$

It is therefore necessary to read both results and ascertain by inspection which is the correct one.

Similarly in extracting cube roots the slide rule gives three possible replies, which must be similarly sorted by inspection.

The operation consists of taking out the slide, reversing the ends, and replacing it. The left-hand end of the bottom scale will now, when the slide is closed, be opposite the right-hand end of the upper scale. If 1 on the part of the slide now uppermost be placed against the number in the upper scale of which it is desired to extract the cube root, that cube root will appear as a similar reading coinciding on the upper scale, and the now upper scale on the slide. As there are two points 1 on the slide, therefore there are two possible settings for it, and out of these two possible settings it will be found that there are three coinciding similar readings, and the correct one must be determined by inspection.

Thus in determining the value of $\sqrt[3]{1.25}$, $\sqrt[3]{12.5}$,

$\sqrt[3]{125}$, $\sqrt[3]{1{,}250}$, $\sqrt[3]{12{,}500}$, and $\sqrt[3]{125{,}000}$, it will be found that if the left hand 1 of the reversed slide be set under 1_{250} on the left hand of the upper scale the numbers 5_{00} and 2_{32} correspond. If now the right hand 1 of the slide be placed under 1,250 on the right hand of the upper scale 2_{32} and 1_{078} will be found to correspond. By inspection it will easily be found that $\sqrt[3]{1.25} = 1.078$, $\sqrt[3]{12.5} = 2.32$, $\sqrt[3]{125} = 5$, $\sqrt[3]{1{,}250} = 10.78$, $\sqrt[3]{12{,}500} = 23.2$, and $\sqrt[3]{125{,}000} = 50$.

SINES AND TANGENTS, &c.—Most flat slide rules in common use have markings on the underside of the scale giving the sines and tangents of any given angle, and conversely the angles corresponding to any given sine or tangent; and a further marking, which enables the logarithm of any given number and the number corresponding to any given logarithm to be found. These vary somewhat in different makes and are best studied with the aid of the makers' description of each particular rule.

METRICAL AND OTHER CALCULATIONS.—Many obvious uses to which the slide rule can be put will occur to the student. For instance a metre = 39.37 in.: if the 1 of the slide be set under 3_{937} of the upper scale the whole becomes a complete table for the conversion of English lineal measurements to metric measurements and *vice versa*. One, in fact, is constantly finding new uses for a slide rule, and as soon as practice has rendered one familiar with its uses it becomes invaluable.

CHAPTER VII

ABBREVIATED METHODS OF CALCULATION (*continued*)

GRAPHIC STATICS

Preference of Draughtsmen for Graphic Methods of Calculation—Graphic Methods Subject to the Usual Disadvantages of Short Cuts—Advantages of Simple Arithmetical Methods—Liability of Graphic Methods to Cause Mistakes in the "Sense" of a Force—Application of Graphic Statics to the Solution of Structural Problems—Resultant, Equilibrant, Component—The Bell-Crank Lever and the Lessons it Teaches—Description of the Statical Board, and its Use—Theorem of the Parallelogram of Forces and its Proof.

PREFERENCE OF DRAUGHTSMEN FOR GRAPHIC METHODS OF CALCULATION.—Arithmetical calculation, even when enormously reduced and simplified by means of the slide rule, is never so popular with draughtsmen as the geometrical method known as graphic statics, which to a very large extent dispenses with the need for figures altogether. For calculating roof trusses, lattice girders, arches and framed structures generally, in which numerous undefined forces act simultaneously in different directions, arithmetical methods, however simple, would often involve such an amount of tedious labour that the judicious adoption of graphic short cuts may be regarded as almost indispensable.

GRAPHIC METHODS SUBJECT TO THE USUAL DISADVANTAGES OF SHORT CUTS.—The student and the practical designer should, however, never forget that graphic methods, like all short cuts, are by no means *invariably* the quickest way by which to arrive at a result. Before employing graphic statics to determine a problem, one should always consider whether by the aid of the slide rule the work cannot be done more or almost as quickly by straightforward arithmetic, for the latter method has many advantages.

ADVANTAGES OF SIMPLE ARITHMETICAL METHODS. —In the first place ordinary arithmetic cannot very well be used without the designer knowing what he is doing, why he is doing it, and what his results mean. He is at any rate able to obtain this important information at any point; but along the graphic short cuts he can but seldom see where he is going, and has little or no means of verifying his ultimate destination.

Figures also can be readily and independently checked at any stage, and if written down systematically and with ordinary care so that their meaning is obvious they can be filed and referred to months and years after the calculations were first made. They can also be fair copied or typed by a clerk, which is a no small convenience in a busy office when copies of calculations have to be submitted to some public authority. But even with careful supervision it is by no means an easy matter to induce the average draughtsman to letter his diagrams fully and clearly enough to enable them to be independently checked without redrawing; and at times of pressure, when the need for careful revision is greatest, they are very apt to be drawn in pencil on any handy piece of tracing paper, which sooner rather than later finds its way into the roll of preliminary sketches, and with them into the waste-paper basket.

Liability of Graphic Methods to Cause Mistakes in the "Sense" of a Force.—It is also necessary when using graphic methods to remember that they form very beautiful labour-saving machines which work just as well backwards as they do forwards. A very common error, which frequently arises in practice, is to ascertain, quite accurately, the amount of stress in a member of a framed structure, but then to mistake its "sense" and to calculate for compression instead of for tension and *vice versa.* The number of ties in roof trusses which are really doing duty as struts, and which are only saved from collapse by a generous factor of safety, is considerable.

The Application of Graphic Statics to the Solution of Structural Problems.—This consists of representing forces acting in the same plane by means of lines; the length of a line to scale representing the amount of a force, its direction representing the direction of the force, whilst small arrow heads are used to indicate which way the force is acting along that direction—or, as it is generally called, the "sense" of the force. For instance, in Fig. 23 a line A B 2 in. long drawn to a scale of 3 cwt. to the inch indicates a force of 6 cwt. acting at 30° from the horizontal. Without the arrow head the diagram might indicate a force acting either from B towards A, or from A towards B. The arrow head makes it clear that it is acting from A towards B.

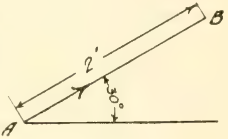


Fig. 23.

There are a few terms frequently used in connection with the representation of forces by means of lines, the meaning of which should be clearly defined.

RESULTANT.—The resultant of two or more forces is that force which, if acting alone, would produce the same result as the two or more forces acting together.

EQUILIBRANT.—The equilibrant of two or more forces is that force which, acting against them, would produce equilibrium or rest. It is therefore equal and opposite to the resultant.

COMPONENT.—Any single force can be resolved into two or more forces whose combined effect is equal to that of the single force acting alone. These two or more forces are called the components of the single force, and the single force is the *resultant* of the two or more *components*.

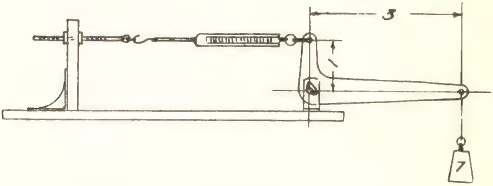


FIG. 24.

THE BELL-CRANK LEVER AND THE LESSONS IT TEACHES.—One of the simplest examples of balanced forces differing both in direction and amount is afforded by the bell-crank lever, which in spite of its simplicity is capable of exemplifying several important principles.

In Fig. 24 there is a bell-crank lever with one arm three times the length of the other. The longer arm is loaded with a weight of 7 lbs. so as to cause a tendency towards rotation in a clockwise direction about the pin centre at the intersection of the long and short arms. This is resisted by the pull of the spring balance on the short arm. When the tightening screw is adjusted so that the spring balance is horizontal the pull it registers is 21 lbs.

↻ CLOCKWISE MOMENT = ANTI-CLOCKWISE MOMENT. ↺

$$7 \times 3 \quad = \quad 21 \times 1$$

The horizontal pull from the balance and the downward pull from the weight balance about the pin, but they are not the only forces acting upon it—there is a reaction from its supports which is very instructive. These supports are cut away in order to enable the pin to be moved vertically, horizontally, or diagonally, by means of a silk loop attached to a sensitive spring balance registering the resistance of the pin to movement in either direction. This is the same thing as registering the reaction which the forces acting upon the pin are calling forth from the supports. The horizontal reaction if measured thus is found to be 21 lbs., and the vertical 7 lbs. There is also one direction and only one in which the silk loop will lift the pin clear in a diagonal direction—the pull necessary being between 22 and 23 lbs.

When the pin is lifted by a vertical pull it will at the same time exhibit a decided horizontal pressure against the vertical side of the slot, and when the pull is horizontal there is an equally pronounced pressure against the bottom of the slot, the diagonal pull being the only one which will neutralise all the pressures on the pin. Obviously the reaction on the pin from two forces is normally in a diagonal direction, and if one only of the two components of that diagonal force be removed or neutralised, then the other component continues to act alone.

If a bell-crank lever with equal arms be tried, as

Fig. 25 (*a*), it will be found that a weight of 7 lbs. produces a pull of 7 lbs., and that the reactions on the pin are 7 lbs. vertical, 7 lbs. horizontal, or a diagonal resultant of nearly 10 lbs. at an angle of 45°.

Precisely the same results are obtained with levers of different shapes (Fig. 25, *a* to *f*) provided that the distance from the direction of each force measured at right angles to the pin remains the same. Whatever be the shape of the lever that is subjected to horizontal and vertical pulls, it will always be found

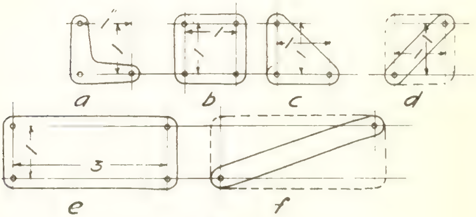


Fig. 25.

that the resulting pressure on the fulcrum consists of forces which are equal to horizontal and vertical pushes of the same amounts and which can be balanced only by a diagonal pull in one particular direction. When the lever takes the form of an inclined bar (*f*) it is easy to appreciate how the horizontal and vertical pulls are combined into one common diagonal resultant which, acting alone along the inclined bar, produces the same result on the pin as do the two pulls which originally generated it. The resultant of two forces or its corresponding equilibrant can only take up one particular direction, and this direction varies with the relative amounts of the forces. The resultant of two forces acting on a balanced bell-crank lever of equal arms is steeper than the resultant of one with dimensions in the proportion of 3 to 1; the one making an angle of 45° with the horizontal and the latter one of about 18°. A few further trials will establish without doubt the fact that both the amount and direction of the diagonal resultant bears a certain fixed relation to the amount of the forces which gave birth to it.

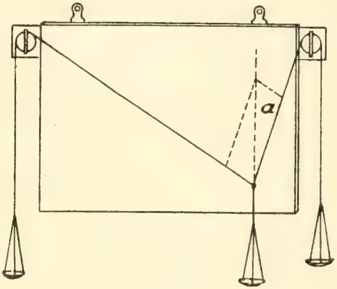


FIG. 26.

DESCRIPTION OF THE STATICAL BOARD AND ITS USE.—These relations can be more accurately determined by what is known as a statical board, as shown in Fig. 26. This consists of an ordinary drawing board at the edges of which are fixed light aluminium grooved pulleys accurately balanced and mounted on centres so that friction is practically eliminated. Over these grooved pulleys run soft flexible silk cords provided with $\frac{1}{2}$-oz. scale pans and weights, or $\frac{1}{10}$-lb. hooks for carrying $\frac{1}{10}$-lb. slotted weights. The whole is hung exactly vertically so that the cords all hang about $\frac{1}{16}$ in. clear of the board. Fig. 27 shows a better but more expensive form of board, round which several pulleys are mounted on blocks which slide freely in grooves, but which can be fixed in any position by means of clamping screws. For ascertaining the direction and amount of two forces, three cords may be used as in Fig. 26 knotted together or tied to a small ring. A sheet of drawing paper having been pinned on the board, the three scale pans are loaded and adjusted until an exact balance is effected; when, if everything is working properly, the ring on being moved in any direction should quickly fly back to its original position. The direction of the cords should then be traced on the drawing paper, that of the resultant being extended as shown in dotted lines in Fig. 26 (*a*). Lengths should then be measured off from the ring along each line representing to some convenient scale the force in each cord, viz., the weights (*including the scale pan*) which it carries. If from the extremity of each measured length, lines be drawn parallel to the adjacent cords in order to form a parallelogram, it will invariably be found that not only is the direction of the resultant or equilibrant the direction of the diagonal of such a parallelogram, but that the length of that diagonal, to the same scale, represents the amount of the resultant or equilibrant.

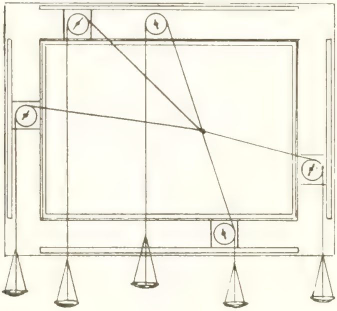


FIG. 27.

Now let the ring be fixed to any position on the board by means of a drawing pin, and any two forces of any convenient amount at any angle greater or less than 180° be applied to it. The forces on the pin are not balanced, and if it be removed the ring will at once fly off to a new position. Let the amount and direction of an equilibrant to those two forces be determined by drawing a parallelogram to scale and extending its diagonal. Let a cord be applied in this direction either by passing it over a pin or preferably over a pulley clamped in the correct position, and loaded with weights corresponding to the scaled length of the diagonal of the parallelogram. If the pin be now removed there should be no tendency of the ring to move off to a new position, which indicates that the forces on it are truly balanced and that the correct equilibrium has been determined. If, however, any other force in any other direction had been applied the ring would, on being released, have at once indicated by moving off to some new position that the forces in it were unbalanced.

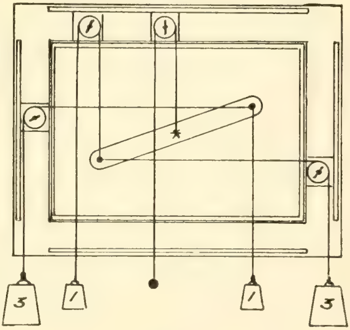


FIG. 28.

The skeleton lever itself can also be placed on the board as in Fig. 28 and balanced under the action of two pairs of forces of which the respective horizontal and the respective vertical forces are equal. If the weight of the lever itself be balanced it will be found that it invariably comes to rest in the position of a diagonal to a parallelogram formed by the four forces.

THEOREM OF THE PARALLELOGRAM OF FORCES AND ITS PROOF.—If all these experiments be repeated with different forces at different angles they should form a volume of striking proof of the accuracy of the important *Theorem of the parallelogram of forces*, viz. :—

If two forces acting at a point be represented in magnitude and direction by the adjacent sides of a parallelogram drawn from that point, then their resultant is represented in magnitude and direction by that diagonal of the parallelogram which passes through the point.

So important is this theorem that in spite of the convincing nature of the simple proofs described above, the following geometrical proof is of value. Consider the triangular beam A B C, Fig. 29, to be pivoted on a pin through the point A, and acted upon by the forces B A and B D. Further, let it be assumed that these forces are neutralised by their equilibrant—which is a force equal in direction and amount to their resultant, and differs from it only in "sense." It is desired to determine the direction and amount of this equilibrant. The force B D obviously tends to cause a rotation of the lever in a clockwise direction, but as the force B A passes through the point A it has no leverage about it, and can exert no tendency to rotate the lever in either direction. The force B D must therefore be balanced by the upward push of the equilibrant of B A and B D. If B D represents to scale the amount and direction of the downward force at B, and A B is

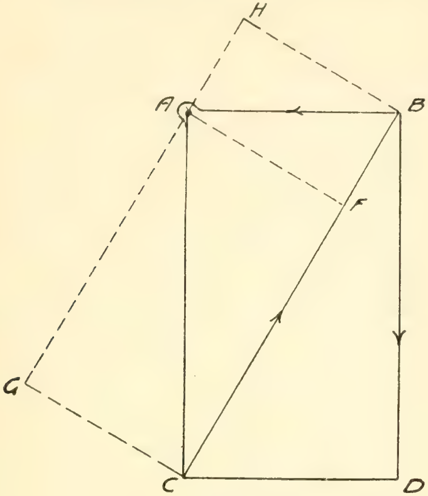


FIG. 29.

the leverage at which it works, then the clockwise moment would be represented by B D × A B, viz., by the *area* of the parallelogram A B C D. A force in the direction C B and of an amount equal to the length C B would have an anti-clockwise tendency, and a leverage of A F, and exercise a moment represented by B C × B H, viz., by the *area* of the parallelogram B C G H. This area, however, is equal to the area B D C A, because one-half of it A B C = one-half of B D C A. Therefore a force represented in amount and direction by C B is acting as the equilibrant of B A and B D, and this force exactly fulfils the conditions of the theorem of the parallelogram of forces.

CHAPTER VIII

ABBREVIATED METHODS OF CALCULATION (*continued*)

GRAPHIC STATICS (*continued*)

Importance of Direction of Bars in a Framed Structure—Theorem of the Triangle of Forces—Theorem of the Polygon of Forces.

IMPORTANCE OF THE DIRECTION OF BARS IN A FRAMED STRUCTURE.—The stress in any bar or member in a framed structure is governed not only by the work it has to do in resisting forces and reactions which are received at its ends, but also by its direction as compared with the direction of those forces.

If a weight be hung vertically by a cord to a spring balance suspended from a hook, as in Fig. 30 (*a*), the reaction of the spring in the balance acts in a direction directly opposite to that of the force of gravity acting upon the weight—the two forming the *pair* of forces necessary to create tension in the cord.

But if the latter be pulled to one side, as in Fig. 30 (*b*), the reaction of the spring balance is not in such a favourable position as before for performing the work it is called upon to do—that of preventing the weight from falling in a vertical direction. It, therefore, has to put forth a greater effort to do the same work, and although it is only pulling up 10 lbs. vertically it indicates a slanting pull of 11.5 lbs.

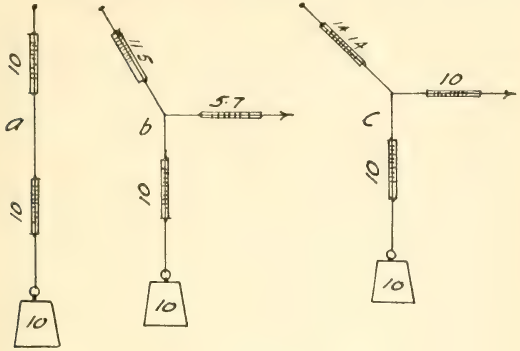


FIG. 30.

If the cord be pulled still more to one side, as in Fig. 30 (*c*), the reading is still higher—14.14 lbs.

The difference made by the direction in which a rod or cord is forced to act can be even more strikingly shown by means of a 2-lb. weight and a piece of ordinary sewing thread. If the weight be hung on a loop of the thread with the two ends of the loop held close together, as in Fig. 31, the weight of 2 lbs., distributed over the two threads forming the loop, is carried easily by the material. If, however, the two ends be moved apart so that the directions

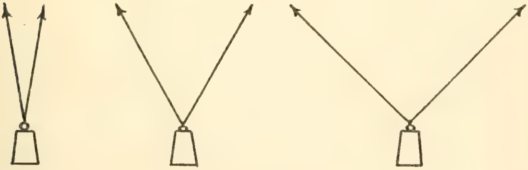


FIG. 31.

of the threads are more and more inclined to their work, the tension in them increases to such an extent that at a certain inclination they invariably break. If the ends of the thread be attached to spring balances, it will be clearly seen that the tension increases as the angle between the two halves of the thread is increased.

If a simple couple made of two laths loosely bolted together at the apex and secured at the foot by a string carrying a spring balance, as in Fig. 32, be loaded by a weight hung from the apex, it will be found that the tension in the tie as registered by the balance varies according to the angle at which the rafters are fixed. The work of the rafters is to push up the loaded apex, and the more they are, by the shape given to the couple, inclined away from that work, the harder they have to push, the greater is the reaction of their feet on the tie, and the greater, therefore, is the tension in the tie.

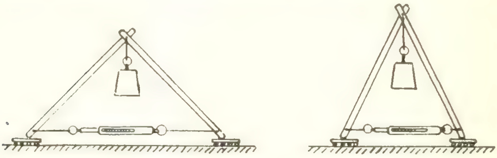


FIG. 32.

THEOREM OF THE TRIANGLE OF FORCES.—Returning to the case of the weight whose downward tendency is resisted by the action of an inclined spring balance, and considering the forces acting at the point where the spring is pulled aside, if a triangle be drawn whose sides are parallel to the direction of those forces and one side of which represents to scale the amount of its corresponding force, it will be invariably found that the lengths of the other two sides to the same scale represent the amounts of the other two forces. This is usually stated in the form of what is known as the *Theorem of the triangle of forces*, viz.:—

"*If three forces acting at a point are in equilibrium and a triangle be drawn having its sides parallel to the direction of the forces, the sides will be proportional to the forces to whose directions they are parallel.*"

The truth of this theorem can be proved by numerous experiments with a statical board, as shown in Fig. 26, or with a board without pulleys, by using light spring balances as shown in Fig. 33, the pulls, and therefore the directions, of the forces being varied by altering the lengths of the cords. It will readily be seen that the triangle of forces is merely one-half of a complete parallelogram of forces, and is formed of three essential parts of the latter.

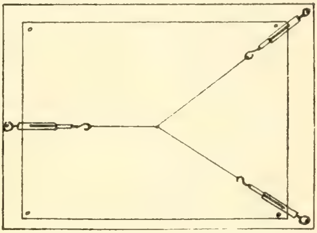


FIG. 33.

The most convenient way of applying this very useful theorem is as follows:—

An outline diagram called the "structure diagram" is drawn as in Fig. 34, showing the structure in skeleton and the direction in which the forces are acting, and each of the *spaces* between each member or force is numbered. One member of the structure is then selected in which the stress is known and a line is drawn parallel to it and representing by its length the known stress to some convenient scale; it is then lettered to correspond, and upon it is built up what is known as the "force diagram." In the present case the vertical part of the cord carrying the weight of 10 lbs. would be taken as a convenient starting point, and a vertical line drawn to a scale of, say, $\frac{1}{5}$ in. to 1 lb., viz., 2 in. long. As this line lies *between* the figures 1-3 in the structure diagram, its *extremities* would be numbered 1-3 in the force diagram and the sense of the stress indicated by an arrow

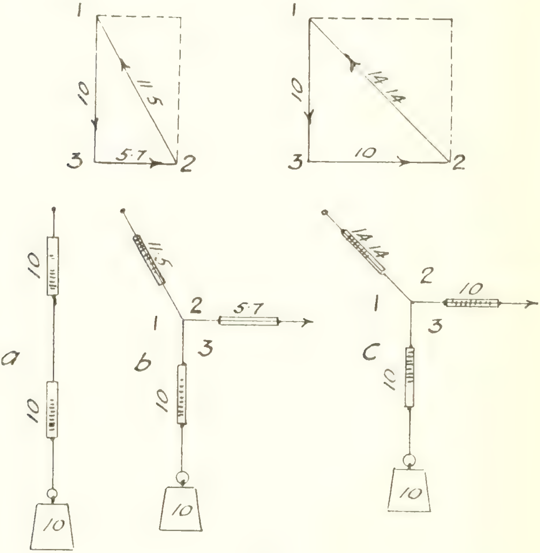


FIG. 34.

head. The remaining forces, of which the direction only is known, are then computed geometrically. Taking any one of them, say the horizontal pull between 2 and 3 in the structure diagram, if the line representing this in the force diagram is to be numbered similarly, then one of its extremities will meet the extremity 3 of the line just drawn. From this point, therefore, a line is drawn of indefinite length, but parallel to the direction of the force between 2 and 3, and its sense is indicated by means of an arrow head. A line parallel to the inclined stress between 1 and 2 in the structure diagram is similarly drawn from the extremity 1, and where this line cuts the line last drawn will be the common extremity 2 of both lines. Its sense having been indicated as before by an arrow head, the force diagram is complete, and if it is correctly drawn the scaled lengths of the lines should exactly represent the several forces acting at the point. It will be noticed that the arrow heads follow round the diagram. This is always the case when they have been correctly drawn and is an extremely useful property.

Only the forces acting at one particular point in the structure have been taken; but if the horizontal and vertical reactions on the hook carrying the balance are added, as shown in dotted lines, then it will readily be seen that the triangle of forces is merely one-half of the complete parallelogram of forces. It should be noted that any one of the forces is necessarily the equilibrant of the other two.

The practical use of the theorem of the triangle of forces will be easily recognised. If it be known that three forces are acting at a point, of which three forces the direction and amount of one is known, then if the direction of the other two are known their amounts can be ascertained ; if their amounts are known their direction can be traced, and if the direction and amount of one is known the direction and amount of the other can be fixed. Further, if the sense of one be known, the sense of both the others can be determined by means of the fact of the indicating arrow heads always following round the force diagram.

Take the very ordinary case of three members in a roof or other framed structure meeting at a point —then if the stress in any one be known, the stresses in the other two can readily be ascertained, and according to whether the known stress be compression or tension, *i.e.*, whether it be pulling from or pushing on the point in question, an arrow head can be placed in the force diagram, which fixes the direction of the other arrow heads in the diagram and at once determines whether the forces which they indicate are tensile or compressive.

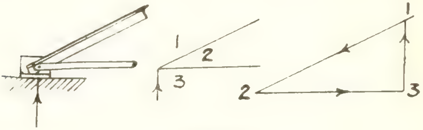


FIG. 35.

In Fig. 35 is shown the shoe of a roof truss, at which point three forces meet, viz., the stresses in the rafter and tie-rod, and the reaction of the abutment from the total load on the roof. It is desired to ascertain the amount and nature of the stresses in the rafter and the rod. The rest of the roof framing may be simple or complex and the loading may be concentrated, distributed, or irregular; that does not concern the question at all so long as the *reaction* is known—the force diagram shown gives all the necessary information. The reaction is pushing upwards, so that the arrow head on line 3-1 is upwards, that on line 2-3 is to the right, indicating a force pulling away from the point or tension; whilst that on line 1-2 is downwards to the left, indicating a line pushing on the point or compression.

THEOREM OF THE POLYGON OF FORCES.—The conditions governing a greater number of forces acting on a point are expressed by what is known as the *Theorem of the polygon of forces*, viz.:—

If a number of forces acting at a point can be represented in magnitude and direction by the sides of a closed polygon taken consecutively, the forces are in equilibrium;

and inversely,

If any number of forces act at a point in equilibrium, and consecutive lines be drawn respectively parallel to and representing in magnitude the said forces, then these lines will form a closed polygon.

The truth of this can be investigated practically by means of numerous experiments with a statical board fitted with several adjustable pulleys, as in Fig. 27, either by arranging a number of forces in equilibrium and ascertaining that the corresponding force diagram drawn to scale forms a closed polygon, or by fixing with a drawing-pin a small ring under forces not in equilibrium, drawing the closed force diagram as far as possible, and then applying to the ring the force represented in magnitude and direction by a line which would close the polygon, and ascertaining that on releasing the ring, the force as represented forms the equilibrant, hitherto absent, of the forces previously unbalanced. The theorem of the polygon of forces is, however, merely an extension of the theorem of the triangle of forces, and its truth is a corollary of the latter.

Consulting the case of the five forces acting on a point as in Fig. 36, according to the theorem, the forces to scale should be represented by a closed polygon. Let one of the forces, say 4-5, be removed —the point is now acted upon by four unbalanced forces.

Take the two forces 1-2 and 1-5, and find their equilibrant 5-2 by drawing a triangle of forces. Their resultant or equivalent would be equal and opposite, viz., 2-5. Ascertain by another triangle a force 4-2, equivalent to the two forces 3-2 and 4-3. Obviously the point can be considered as being acted upon by the two *equivalents* of the four forces, and therefore the equilibrant of these two equivalents is the general equilibrant of the four forces. Equally obvious is it that the equilibrant of the two equivalents is the closing line of the triangle of forces 4-2-5, which also forms the closing line of the polygon. It will be seen that any line in a polygon of forces is necessarily the equilibrant

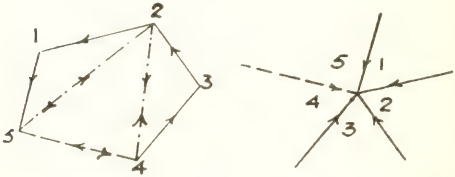


FIG. 36.

of all the other forces. It will also be seen that the useful property of the arrow heads in following round the triangle of forces holds good also in the polygon of forces ; so that the nature of the stress in any bar acting at any joint of a framed structure can be readily obtained from the force diagram, whatever may be the number of forces acting at the point.

CHAPTER IX

ABBREVIATED METHODS OF CALCULATION (*continued*)

GRAPHIC STATICS (*continued*)

The Funicular or Link Polygon: Its Utility in Practical Design—Investigation of the Stresses in a Model Polygon—The Reason for the Omission of Arrow Heads from Stress Diagrams—Peculiarities of a Frame of Unbraced Links and its Stress Diagram—Determination by the Link Polygon of the End Reactions of Irregularly Loaded Beams—Determination of Centre of Gravity by the Link Polygon—Examples of the Use of the Link Polygon.

THE FUNICULAR OR LINK POLYGON AND ITS UTILITY IN PRACTICAL DESIGN.—One of the most useful geometrical tools is what is known as the funicular or link polygon. By its aid the centre of gravity of any irregular figure or any number of irregular forces can be very quickly found without calculation, together with the reactions which those forces would generate on the bearings of a beam or roof truss. A consideration of the forces acting in a link polygon also enables the operation of building up stress diagrams to be traced with considerable lucidity.

INVESTIGATION OF THE STRESSES IN A MODEL POLYGON.—Its properties are best studied in a practical manner by hanging a light lath on two spring balances, and attaching to it three or more loops of string carrying light spring balances and loaded irregularly, as in Fig. 37. If these be hung just clear of a vertical drawing board upon which is pinned a sheet of drawing paper, the structure diagram can be easily traced, as in Fig. 38 (*a*). All the stresses and reactions for any loading of the model being known through the medium of the different spring balances, the accuracy of diagrams drawn for the purpose of calculating those stresses and reactions can be readily tested. The arrangement forms an elastic frame in which the tendency towards alteration of shape due to any variation of load can be seen.

As soon as the frame has come to rest under any given arrangement of loads, the stresses in the various members are determined by the direction which they have taken up with regard to the load, just as if they had been originally fixed in those directions in the same way that the members of a roof truss or a lattice girder are fixed by the original design.

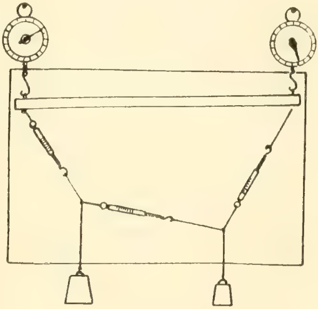


FIG. 37.

If all the spaces in the structure diagram were numbered, a complete diagram of stresses could at once be drawn; but for the purposes of investigation it is better to draw separate force diagrams for each angle of the frame, and to see how they combine to form the complete stress diagram.

Taking the forces at the right-hand side of the lath in Fig. 38 (*b*), the upward reaction 1-2 and the inclined force 2-0 are drawn parallel to the direction of those forces, but alone they do not form a closed figure. The angle of the frame, however, is in equilibrium, otherwise it would move, as it is quite free to do; and the conditions of equilibrium demand a closed figure. The closing line is a force acting between 0-1, and the placing of arrow-head lines representing the two other forces establishes the fact that the missing force is one pushing on the point; in other words, that there is a compressive force acting along the lath of an amount represented by the length 0-1. If a compression balance be introduced in the lath as shown this would be the amount which it would register. If a similar diagram be drawn of the forces acting at the lower right-hand angle of the frame, Fig. 38 (*c*), it will result in a closed triangle in which the arrow heads show that all the forces are pulling away from the point, *i.e.*, that all the strings are in tension.

The force diagrams, Fig. 38 (*b*) and (*c*), can be easily combined as (*d*), the line 2-0 being common to both. The diagram (*e*) for the forces acting at the lower left-hand corner of the frame also forms a closed figure and can be combined with the combined diagram (*d*), the line 0-3 being common to both. The forces acting at the left-hand end of the lath are the reaction 4-1, the inclined pull 0-4, and the compression in the lath. These form a figure identical with the triangle 1-4-0, and the combined stress diagram (*f*) for the whole frame is now complete. It should be observed that it can be entirely built up from either of the lines representing the end reactions, or the loads, provided that the *direction* of

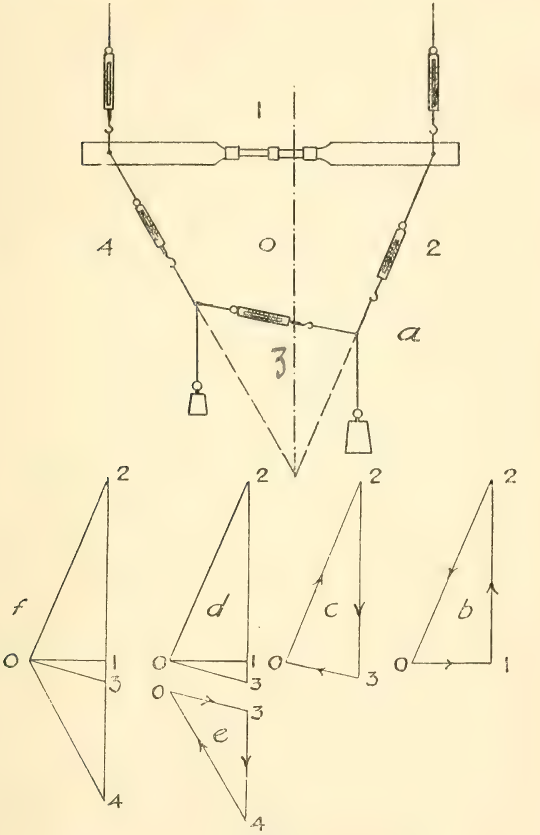


FIG. 38.

the members is known. It merely consists of the force diagrams of the forces acting at each angle or joint of the frame superimposed upon each other. At no joint are more than three forces acting, so that each force diagram is a triangle. Had the frame been braced internally more than three forces would have met at any point, and the force diagrams would have been polygons, and all the lines in the combined stress diagrams would not have met at the common point of intersection O.

THE REASON FOR THE OMISSION OF ARROW HEADS FROM STRESS DIAGRAMS.—It should also be noted that the arrow head showing in any one diagram the sense of the force in any member is reversed in the adjacent diagram, so that the member is denoted as pulling on one joint of the frame with one end in one direction and pushing on the adjacent point with the other end in the contrary direction. As a matter of fact it is really the joints of the frame which pull or push on the members and call forth equal and opposite reactions from them, and it will be readily understood why, when force diagrams are superimposed on each other to form a stress diagram, the arrow heads are omitted. It would be difficult always to bear in mind the difference between action and reaction, and two arrow heads pointing towards each other on a tension member would be very liable to be taken as indicating compression and *vice versa*. When, therefore, it is desired to ascertain whether the stress in any member is compression or tension a diagram of the forces acting at one end of it (*i.e.*, at one joint in the frame), is either drawn or extracted from the complete stress diagram of the whole frame. Arrow heads on such a diagram started from some known force will show at once whether the particular member in question is pushing on the point or pulling away from it.

PECULIARITIES OF A FRAME OF UNBRACED LINKS AND ITS STRESS DIAGRAM.—In practice stress diagrams are not built up from superimposed force diagrams; the known loads and reactions are laid down to scale, and the stresses in the members are set out by drawing lines parallel to the direction of the members, and if necessary using their points of intersection as starting points for fresh lines. In the case under consideration the vertical loads 2-3 and 3-4 would first have been laid down, and from the points 2 and 3 lines would have been drawn parallel to 2-0 and 3-0. A line from 4 would pass through the point of intersection, and line from 0 parallel to 1-0 would cut the vertical line of loads into two portions 2-1 and 1-4, which would represent the reactions at each end of the lath.

One of the peculiarities of a frame of unbraced links is that, being free to change its shape under any alteration of loading, the *inclination* of its members is determined by the loading. For this reason the stress diagram can be built up from the loads alone, the reactions being automatically arrived at. Another peculiarity is that, owing to the fact that all the force diagrams which constitute the stress diagram are necessarily triangles with one side common to each two adjacent diagrams, all the lines in the stress diagrams have a common point of intersection 0, generally called the pole. A further peculiarity, the reason of which will be explained later, is that if the direction of the end links of a frame under vertical loading be prolonged until they intersect, the point of intersection is vertically under the centre of gravity of the loads.

DETERMINATION BY THE LINK POLYGON OF THE END REACTIONS OF IRREGULARLY LOADED BEAMS.—It is very frequently necessary to ascertain the end reactions and the centre of gravity of a number of parallel vertical loads on a beam or roof. This somewhat tedious process can be performed very rapidly and accurately by *imagining* the loads as hanging upon a loosely-jointed unbraced frame of links. A B, Fig. 39, represents a girder carrying several loaded joists—it is necessary to ascertain the reactions at A and B and the centre of gravity of the loads. The difference due to the weight of the girder and its individual load can be treated separately or added as forces acting at their centres of gravity. If the loads were hung from the girder on some unbraced frame of loosely-jointed links, the stress diagram of that frame would consist of the vertical loads and reactions and a number of inclined lines, generally called vectors, from the ends of the load lines intersecting at a common point or "pole" O. The loads can be imagined as hung from any one of an infinite number of frames of links, each one of which would have its own particular corresponding stress diagram. The vertical lines would, of course, be the same in all the stress diagrams, the only difference between them being in the length and direction of the inclined lines, viz., *in the position of the pole*. The stress diagram corresponding to *one* of the innumerable imaginary frames of links can be at once drawn by arbitrarily marking a pole anywhere conveniently near the plotted load line and drawing vectors to it. The usual process is therefore inverted, and instead of the stress diagram

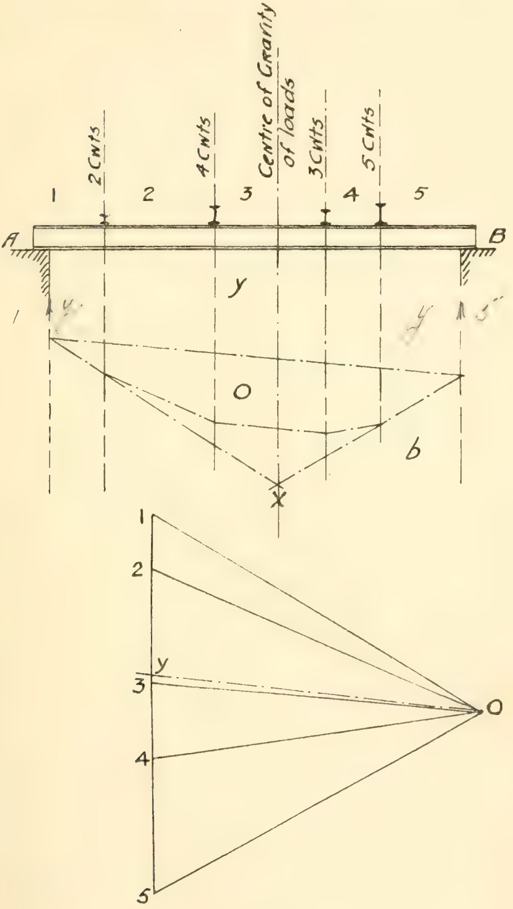

2 Cwts
4 Cwts
Centre of Gravity of loads
3 Cwts
5 Cwts
1
2
3
4
5
A
B
y
O
b
X
1
2
y
3
4
5
O

FIG. 39.

being drawn parallel to the structure diagram, *the structure diagram is drawn from the stress diagram.*

Vertical dotted lines are drawn through the direction of the loads, and lines parallel to the radiating lines of the stress diagram are drawn to intersect upon them (Fig. 39). The direction of the imaginary lath or compression member is obtained by joining the ends of the two outside links. This is not necessarily horizontal, it probably will not be. From the pole O a line parallel to this direction is drawn to cut the line of loads, the latter being thus divided up into the two reactions 1 y at A and y 5 at B which are together equal to the total load 1-5.

DETERMINATION OF CENTRE OF GRAVITY BY THE LINK POLYGON.—To obtain the position of the centre of gravity of the loads, it is only necessary to extend the direction of the end links in the structure diagram to intersect at X—a vertical line through that point will intersect on the beam at the centre of gravity of the loads. The reason why the end links always intersect in a line passing through the centre of gravity of parallel forces is as follows:—

In a link polygon all the forces, loads, and reactions between the ends combine to form at the ends two sets of three forces, one of which forces in each set of three is the reaction. When three forces are in equilibrium any one of them forms the equilibrant of the other two—so that the two sets of three forces can be further reduced to the stresses in the two outside links. Any equilibrant of these two forces must pass through the point of intersection of their directions. In other words, all the forces in the frame *could*, therefore, be balanced by one force passing through a known point. As the equilibrant of the vertical loads must itself be vertical, the direction of the equilibrant must be a vertical line between the point of intersection and the beam, and where such an equilibrant could be applied to the beam must be the centre of gravity of the loads.

EXAMPLES OF THE USE OF THE LINK POLYGON.—The link polygon can be applied to the determination of the centre of gravity of an irregular figure by dividing the figure up into constituent parts, the areas and centres of gravity of which are known, and then treating the parts as if they were loads or forces acting in parallel directions.

Let Fig. 40 represent the section of a retaining wall with a battered face and stepped back. It is required to ascertain the *horizontal* distance of the centre of gravity from the toe D. The section can be conveniently divided up into three rectangles and a triangle, whose centres of gravity can be graphically fixed at once. Vertical lines are drawn through these centres, and the spaces between the lines numbered. The areas of the parts are then drawn to scale as a load line, and a pole is selected anywhere conveniently near, and joined to the points on the load line by vectors. In the structure diagram the outside

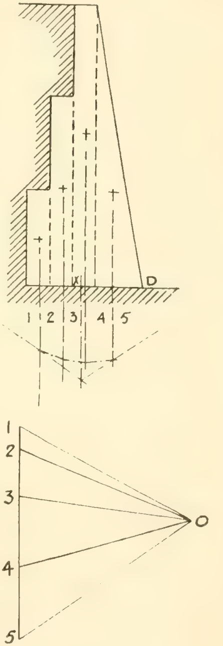


FIG. 40.

lines will be of indefinite length, because no points are fixed on which the reactions (viz., the attachments of the imaginary link frame) are to be calculated. No reactions, however, are required ; all that is required is the inclination of these two outside lines towards each other, and this is fixed by the stress diagram.

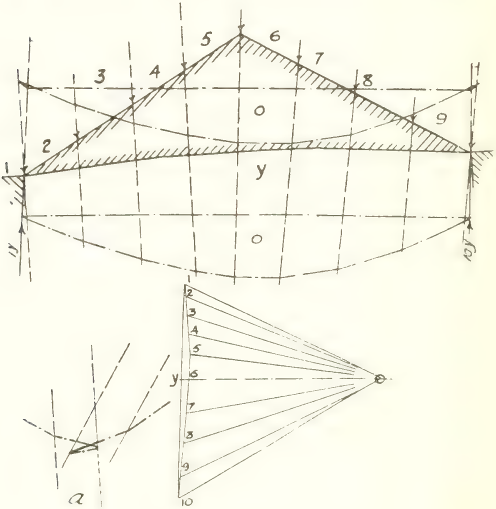


FIG. 41.

A vertical line through their point of intersection cuts the base of the wall at a distance X D from the toe.

The link polygon can also be used to determine the reactions of forces which are not parallel.

The roof truss in Fig. 41 carries vertical loads on the left-hand side due to the weight of the roof covering etc., and on the right-hand side it carries the inclined resultants of similar loads acting in conjunction with a horizontal wind.

The various loads are set down to scale, and a line joining the extremities of the bent line which results will represent the sum of the *two* reactions. A convenient pole is chosen, vectors drawn, and the structure diagram is completed in the usual way. A line parallel to the closing line cuts the inclined line of reactions at Y into two portions, 10-Y and 1-Y.

When the forces are not parallel it is not always possible to draw a straightforward structure diagram, and the latter may have to be doubled back upon itself as in the upper diagram of Fig. 41. This does not, however, affect the accuracy of the results, and the inclination of the closing line will be found to be the same wherever the diagram be drawn, even though the position selected for the starting point should involve such a complication as is shown at Fig. 41 (*a*).

PART II

BEAMS AND GIRDERS

CHAPTER X

BENDING MOMENTS

Beams and Cantilevers—Bending Moments and Moments of Resistance Defined and Compared—B.M. the Attacking, M.R. the Resisting Force—B.M. always Equal to M.R. in Unbroken Beams—Resemblance between the Action of a Bell-Crank Lever and the B.M. and M.R. in a Cantilever—Bending Moment Simply a Resisted Turning Moment—Two Equal Bending Moments Necessary to Produce one Stress—Universal Counterpoise in Quiescent Structures the Basis of all Static Theory—B.Ms. easier to Study in their Primary Form of Turning Moments—Graphic Representation of B.Ms. in a Cantilever.

BEAMS AND CANTILEVERS.—In many text-books the term beam is used to cover all classes of supports carrying loads and spanning an open space; what is generally known as a cantilever being defined as a beam fixed at one end and free at the other. In the following pages, however, the term beam is confined to those members which are supported or fixed at both ends, and the term cantilever is used in its more customary sense.

BENDING MOMENTS AND MOMENTS OF RESISTANCE COMPARED AND DEFINED. — The term "moment" was defined in Chapter IV. as being a relative measure used and prepared for the purposes of comparison of the true efficiency of a force, viz., its amount multiplied by its mechanical advantage or leverage. Any force which, in attacking a beam or structure, tends to bend it, is, when multiplied by its mechanical advantage or leverage, termed a Bending Moment.

B.M. THE ATTACKING, M.R. THE RESISTING FORCE.—The internal resistances by which the fibres of the beam offer opposition to this bending tendency are, when similarly multiplied by their mechanical advantages or leverages, termed the Moment of Resistance of the beam. It must be kept clearly in mind that the two are absolutely distinct—the one is the attacking and the other the resisting force.

B.M. AND M.R. ALWAYS EQUAL IN UNBROKEN BEAMS.—If the bending moment is too great, the beam breaks; but in unbroken beams the only connection between the two is like that of action and reaction—*they are always equal to each other.* A small bending moment calls up a small opposing internal reaction consisting of stresses acting at certain leverages, the stresses being only sufficient to make up a total moment of resistance just equal to the bending moment. A larger bending moment demands a proportionately larger moment of resistance, and as the leverages at which the fibres must work are permanently fixed by the shape of the beam and cannot increase, the difference must be made up by an increase in the stresses until the total moment of resistance equals the bending moment. A still greater bending moment involves still higher stresses, and so on. The fact that the moment of the external loads on the beam is always equal to the moment of the internal resistances enables the two to be set down in the form of an equation, Bending moment (B.M.) = Moment of resistance (M.R.). From this any unknown factor can be determined if the rest of the factors forming the equation be known; but the essentially distinct nature and functions of the two attacking and defending forces *must* be always remembered.

Resemblance Between the Action of a Bell-Crank Lever and the B.M. and M.R. of a Cantilever.—From the experiments on bell-crank levers described in Chapter VII., it will be obvious that if the bell-crank lever shown in Fig. 42 be pivoted at B, the clockwise turning moments of the weight W would be W × 18 in. The anti-clockwise turning moment, which is equal to and resists this, is the pull at A × 6 in. The greater the turning moment of W the greater must be the equalising pull at A. The experiments also show that this clockwise moment cannot act without producing a horizontal pressure on the pin at B equal to the pull at A (the vertical pressure at B will be dealt with later in conjunction with vertical shear), and also that a rectangular lever A B C D would act in exactly the same way as the bell-crank lever A B C.

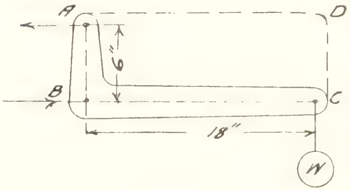


Fig. 42.

If the lever had been pivoted at A it would have required a balancing *push* at B and involved a *pull* on the pin at A of the same amount.

A beam balanced over a log or pivot like a child's see-saw may be regarded as a double cantilever, as in Fig. 43, and therefore as consisting essentially of a pair of bell-crank levers, the forces which tend to bend it across the pivot at B, by pulling it apart at A and by crushing at B, being the turning moments of the weights W and W.

It was shown in Chapter II. that what is known as tension or compression of any given amount can only take place under the action of a *pair* of forces *each equal to that amount*, so that a tensional stress on the little strap A can only take place under the action of the *pair* of forces at A induced by the contrary moments of the *two* weights W and W, and

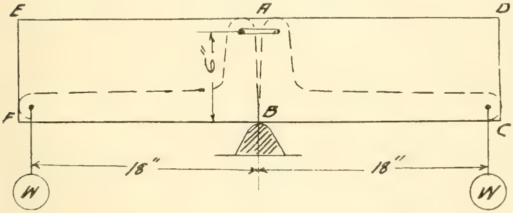


FIG. 43.

it is equal in amount to the force required to balance *one* of them, not both. Also, that however strong the strap at A may be, it would not be alone sufficient to prevent bending unless the fibres at B were strong enough to resist crushing under the action of the two pushes which take place there simultaneously with the two pulls. It will now be seen that whilst the beam remains unbroken the bending moment at any point is precisely the same as the turning moment at that point, and that the moment of resistance is equivalent to the action of the balancing forces in a bell-crank lever—one arm of which is the effective depth of the girder.

BENDING MOMENT SIMPLY A RESISTED TURNING MOMENT.—The fact that bending moments are identical with resisted turning moments is important and should be grasped clearly. If a plank be laid across a log it bends slightly under the action of its own weight. With a child seated at each end it bends more, and with two children at each end it bends still more. The weight of one-half of the plank, however, cannot bend it across the log except by means of the assistance afforded by the weight of the other half—similarly the bending moment set up by one or by two children *must* be balanced by an equal and opposite bending moment at the other side of the log, or it would act as a turning moment and not as a bending moment at all. An equal and opposite reaction is indispensable to every bending moment before it can operate to stress fibres. The first action of a load upon a body or structure is to set up a turning moment tending to cause the body or structure to rotate. If, and only if, it is prevented by means of an equal and opposite reaction from thus turning the structure, then it bends it. If it cannot set up a turning moment then it sets up a bending moment instead. Even if the plank had been built into a wall instead of being balanced about a log, bending could not be caused unless the wall were strong enough to supply the necessary reaction. If the wall were loose and weak the turning moment of a weight on the end of the log would have torn it apart and caused turning instead of bending.

Now consider the see-saw to be balanced and to be carrying three children and two-thirds of the length of the plank on the right of the log, and on the left one-third of the plank and a woman sitting on the end. What is the bending moment at the log? There are apparently quite an embarrassing number of bending moments there—three children, a woman, and two unequal lengths of plank all acting at different leverages. It will be remembered that although a pull of, say, 4 lbs. in a cord is impossible without an opposing pull of 4 lbs., we speak of the result of both as a 4-lb. tension—taking the necessary reaction for granted.

TWO EQUAL BENDING MOMENTS NECESSARY TO PRODUCE ONE STRESS.—It takes two equal and opposite pulls to produce tension, and we measure one and call that the tension. Similarly although a bending moment of, say, 4 ft.-lbs. is impossible without an opposing moment of 4 ft.-lbs., it is in the same manner alluded to as a bending moment of 4 ft.-lbs., without any reference to the indispensable opposing moment. It takes two equal and opposite bending moments to produce stress at any point, but we calculate one only and call that the bending moment at the point. To express the bending moment at any point of a beam or any other structure *all* the forces on one side are taken into account and *all* the forces on the other side are neglected. The bending moment at the log is the bending moment coming from one side *or* the other. It does not matter which, because they must be equal or the plank would move; but it is not *both*. On the one side of the log there are the moments set up by the weight of two-thirds of the plank acting at its centre of gravity and that of three children acting at different distances, the whole forming one combined tendency to bend the plank at the log; on the other there are only two factors, the weight of one-third of the plank acting at its centre of gravity (one-sixth of the length of the plank) and that of the woman acting at twice that distance, forming together one combined tendency to bend the plank at the log, and *exactly* equal to that produced on the other side of the log. Obviously the simpler bending moment to calculate would be that coming from the left. In practice it would be better to calculate both as a check, but that is not arithmetically necessary. Take the case of a beam unequally loaded and projecting over one bearing and loaded on the projecting end. The calculation of the bending moment at that bearing is in this case *only* possible from one side, because the cantilever end only picks up from amongst the numerous bending moments and reactions on the other side just enough to equalise itself.

UNIVERSAL COUNTERPOISE IN STILL STRUCTURES.—It is very necessary to bear in mind the essential condition of equilibrium, viz., that all the moments on one side of a point must be together equal to all contrary moments on the other side. Anything which is not moving is in equilibrium in every part. At any point the turning moments caused by *all* the forces, loads, stresses, and reactions on one side are exactly equal to all those on the other.

It is instructive to lay one's hand on one point of some large complicated structure, like a great lattice bridge, and try to realise this really wonderful fact. There are loads, forces, stresses, and reactions to the right in bewildering profusion—great and small—simple and complicated, and the same on the left. But however numerous, however great, and however complicated they may be, their turning moments on the one side balance the turning moments on the other and balance exactly, otherwise there would be movement under one's hand and not rest. It is difficult to realise this marvellous universal and perpetual state of counterpoise and balance. It is such a simple and everyday matter that we are apt to overlook the wonder of it. But it is worth while making an attempt to realise it, because it forms the basis of the science of structural mechanics, and only by means of it is it possible to measure the stresses in actual structures or to forecast and predetermine those in structures which we intend to build.

BENDING MOMENTS EASIER TO STUDY IN THEIR PRIMARY FORM OF TURNING MOMENTS.—Bending moments can be both studied and graphically expressed much more easily by considering them in their primary form of turning moments without reference to the moments of resistance whose reaction turns them into bending moments.

GRAPHIC REPRESENTATION OF BENDING MOMENTS IN A CANTILEVER.—The simplest bending moment to follow is that of a cantilever loaded at one end. In Fig. 44 is a cantilever 4 ft. long, and loaded with a weight of 1 cwt. at one end. The bending moment at any point due to that load is the moment which the load

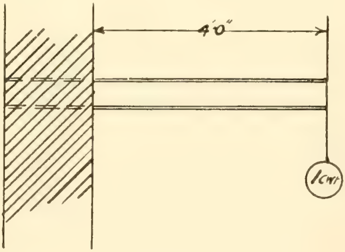


FIG. 44.

exercises, tending to cause rotation. At the wall end A, as a pivot it is 4 ft.-cwt., at 1 ft. from the wall it is 3 ft.-cwt., at the centre it is 2 ft.-cwt., at 3 ft. from the wall it is 1 ft.-cwt., and at the point of application of the load it is nil. This suggests at once a graphic method, Fig. 45, by which unnecessary calculation is saved. If to some convenient scale, say $\frac{1}{2}$ in. to 1 ft.-cwt., the B.M. at A were set off as A *f*, and *f* E joined by a straight line, then the calculation of all intermediate bending moments can be saved; for from the well-known principle of similar triangles, lines drawn from any point parallel to A *f* such as B *g*, C *h*, or D *i*, would bear the same proportion to A *f* as E B, E C, or E D bear to E A. But the turning or bending moments are proportional to the distances E D, E C, E B, and E A, so that the lengths of the lines D *i*, C *h*, etc., represent the bending moments at those points, *measured to the same scale as A f.*

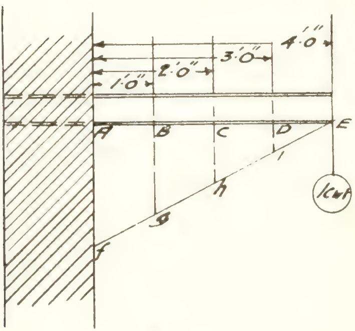


Fig. 45.

Now let another weight be added as in Fig. 46, and let another triangle be drawn to represent the bending moments due to it. It does not matter whether it is drawn with one side parallel to A B as shown in dotted lines or in any other way so long as *f g* represents the bending moment at A and the triangle tapers to nothing at X. The most convenient way of drawing it would obviously be to superimpose it on the triangle AfB, because then the combined moment at any point p due to both weights could be measured off from p to p'' in one operation instead of being collected from two measurements pp' and $p'''p''''$. Fig. 47 shows the same process extended to several unequal weights, and the ease with which the bending moment xy at any point can be obtained is obvious.

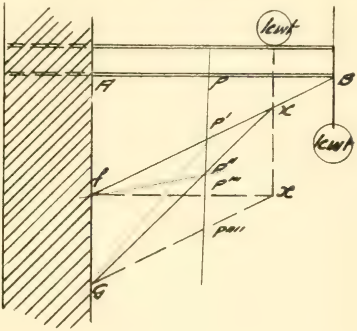


FIG. 46.

It will be seen that the more loads there are the more the line of the bending moments approximates to a curve. In the case of a very large number of equal loads the curve can hardly be distinguished from a parabola. Given a sufficient number of equal loads the line is practically a parabola. In fact this method of indicating bending moments will probably be recognised as one of the well-known methods of drawing parabolic curves.

A distributed load like the weight of a cantilever of uniform section may be regarded as consisting of an infinite number of evenly distributed equal concentrated loads of which the curve of bending moments is a parabola; and for any equally distributed load the line of bending moments can

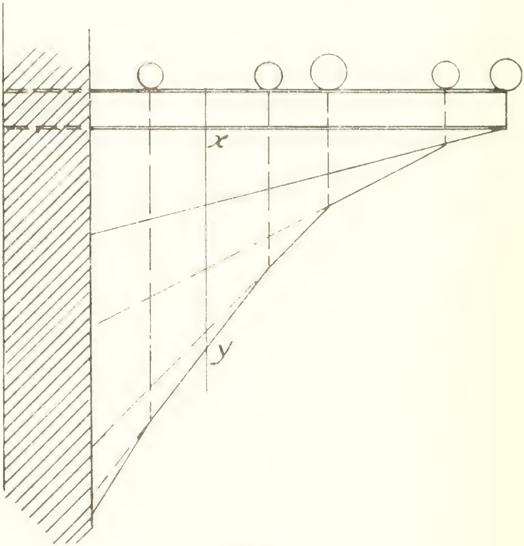


FIG. 47.

be, if necessary, traced at once as a parabola by an instrument.

CHAPTER XI

BENDING MOMENTS (*continued*).

Moments on Cantilevers Identical with those on Beams—Graphic Representation of Bending Moments on Beams Due to Concentrated Loads—Construction of Formula for Concentrated Load—Bending Moments Due to Two or More Loads—Parabolic Curve of Distributed Loads—Construction of Formula for Distributed Loads—Flat Parabolic Curves Practically Identical with Segments of Circles—Combined Diagrams for Distributed and Concentrated Loads—Method of Ascertaining the Length of Plates in Girder Flanges by Finding Points in a Parabola—Formula for Ascertaining Length of Plates in Girder Flanges—Calculation of Bending Moments on Balanced Cantilevers—Girders with Fixed Ends and Continuous Girders—Points of Contrary Flexure—Reaction of Supports of Continuous Beams—The Use of the Funicular Polygon as a Diagram of Bending Moments.

MOMENTS ON CANTILEVERS IDENTICAL WITH THOSE ON BEAMS.—The essentially identical nature of the bending moments on cantilevers and those on beams can be readily appreciated by comparing the case of a balanced cantilever, A B in Fig. 48, extended over the bearing B (where the conditions are essentially those of a simple see-saw), with the case of a beam loaded with a concentrated load, as in Fig. 49, where the conditions are essentially those of the same see-saw turned upside down. If the reader finds any difficulty (owing to preconceived ideas as to gravity) in grasping this, the double cantilever in Fig. 48 should be imagined as laid on its side and acted upon by lateral forces, the bearing B being a post driven into the ground. Then it is quite easy to mentally step round to the other side of the beam and view it in a reversed position, regarding the reaction at the post as the load. It will be seen that the conditions, leverages, and bending moments are precisely the same in both cases, and it will

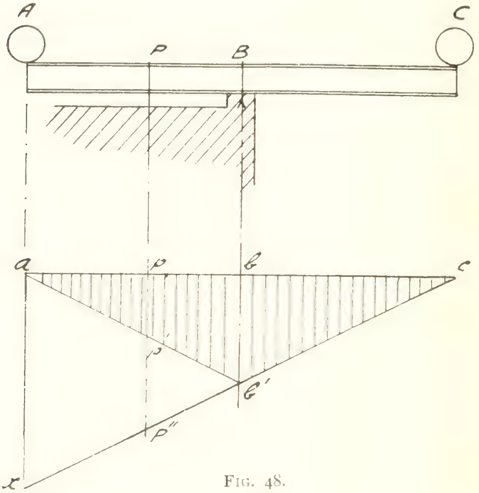


FIG. 48.

also be obvious that the load in Fig. 49 calls up the *reactions* at the bearings, which in turn tend to set up bending moments.

GRAPHIC REPRESENTATION OF BENDING MOMENTS ON BEAMS DUE TO CONCENTRATED LOADS.—If the weight C on the double cantilever be considered as tending to press downwards on B using the weight A as a fixed pivot, the leverage at which it acts will be A C and the reaction at B will resist this at a leverage equal to A B, *i.e.*—

$$C \times AC = B \times AB.$$

$$\therefore\ B = \frac{C \times AC}{AB}$$

$\therefore$ B = 2 C, as is obvious by inspection.

This can be shown graphically as before.

ac represents the span and ax the moment C × AC; then any ordinate pp'' of the triangle axc will represent the bending moment at any point P. The contrary bending moment A × AB will also be represented by ax, and the reverse bending moments at any point P will be represented by the corresponding ordinate $p'p''$ of the triangle axb'. This triangle superimposed upon the triangle axc will in fact slice off from the latter the contrary bending moments due to the reaction at B, and the shaded part $ab'c$ which

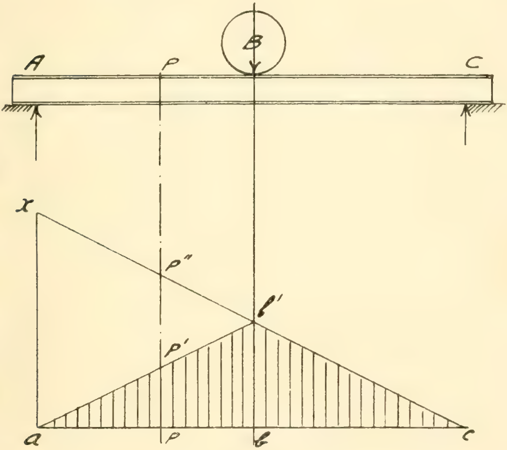


FIG. 49.

remains will represent the net bending moments, and at any point P the net bending moment will be $p p'$. The diagram of net bending moments consisting of the triangular bending moment diagrams of the two half cantilevers.

CONSTRUCTION OF FORMULA FOR CONCENTRATED LOAD.—The maximum B.M. on a span L due to a concentrated load W in the centre will therefore be either reaction $\left(i.e., \frac{W}{2}\right) \times \frac{L}{2} = \frac{W L}{4}$.

BENDING MOMENTS DUE TO TWO OR MORE LOADS.—The case of the beam A B, Fig. 50, loaded with two weights C and D can be solved in practically the same way. The ordinate $a x$ equals the moment

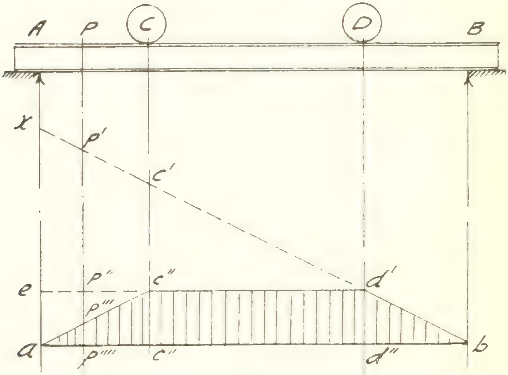


FIG. 50.

of the reaction at B (R b) × A B ; and from the triangle $a x b$ the reverse bending moments due to the weights C and D are sliced off. The length $c x$ equals the moment of D × A D, and the triangle $a c'' e$ represents those due to C—the ordinate *ea* being equal to the moment C × A C. The net bending moment at any point P is made up as follows: First comes the anti-clockwise effort of the reaction at B acting at a leverage of B P as in a reversed cantilever. This would, in a diagram of cantilever bending moments, be represented by the ordinate p'-p''''. From this must be deducted the contrary clockwise effort of the weight D, *i.e.*, D × D P. This would similarly be represented by the ordinate $p'p''$, and is automatically deducted from the moment R B × B P by the method of drawing the diagram. Similarly the ordinate $p''p'''$ also slices off the necessary deduction to represent the clockwise moment C × C P.

PARABOLIC CURVE OF DISTRIBUTED LOADS.—The greater the extent to which the total load is subdivided, the less pronounced do the peaks in the B.M. diagram become, as in Fig. 51, until with a load which is perfectly distributed, like the weight of the girder itself when of uniform section throughout, as in Fig. 52, the line subsides into a parabolic curve, the method of drawing it being, as was the case with cantilevers, one of the methods of drawing a parabola.

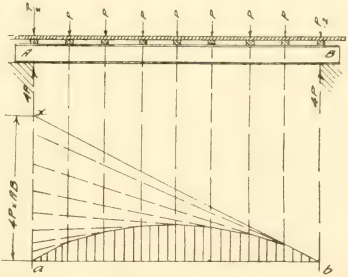


FIG. 51.

CONSTRUCTION OF FORMULA FOR DISTRIBUTED LOADS.—The maximum B.M. due to a total load

W equally distributed over a span L, as in Fig. 52,

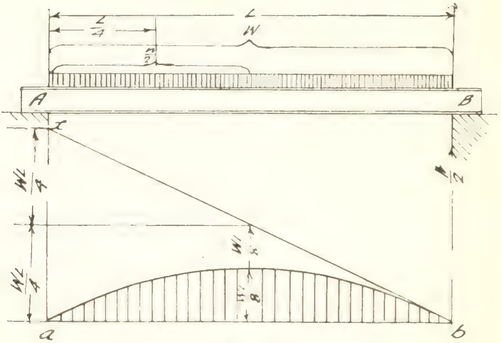


FIG. 52.

will be at the centre. It will consist of either reaction, *i.e.*, $\frac{W}{2} \times \frac{L}{2}$ less the reverse B.M. due to the load on half the span acting at its centre of gravity, *i.e.*—

$$\left(\frac{W}{2} \times \frac{L}{2}\right) - \left(\frac{W}{2} \times \frac{L}{4}\right) = \frac{WL}{4} - \frac{WL}{8} = \frac{WL}{8}.$$

If the load had been expressed in the form of w per foot-run of the span the formula would be :—

$$\left(\frac{wL}{2} \times \frac{L}{2}\right) - \left(\frac{wL}{2} \times \frac{L}{4}\right) = \frac{wL^2}{4} - \frac{wL^2}{8} = \frac{wL^2}{8}.$$

FLAT PARABOLIC CURVES PRACTICALLY IDENTICAL WITH SEGMENTS OF CIRCLES.—It is useful to note that when the central ordinate of a parabolic curve does not exceed one-fourth of the base, the curve cannot, for practical purposes, be distinguished from a segment of a circle. Therefore, if the scale of foot-tons or other units adopted for the bending moments be selected so that the central ordinate does not exceed this proportion of the span, the curve can be drawn as a segment of a circle with a pair of compasses.

COMBINED DIAGRAMS FOR DISTRIBUTED AND CONCENTRATED LOADS.—Fig. 53 shows the very ordinary case of a girder carrying a distributed load due to its own weight, and to the live and dead load of a bay of flooring, and in addition three unequal concentrated loads from cross joists at various points. If the live load be doubled to give the equivalent dead load, the B.M. diagrams can be readily drawn with compasses by selecting such a scale of inch-

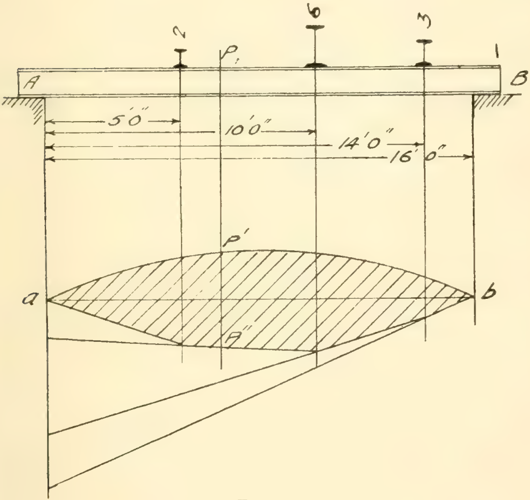


FIG. 53.

tons as will give a central ordinate $\frac{WL}{8}$ or $\frac{wL^2}{8}$, due to the distributed loads, which will not exceed $\frac{L}{4}$. The line of moments due to these loads can then be drawn at once as a segment of a circle on one side of the line, and the irregular line of moments due to the concentrated loads traced on the other side of

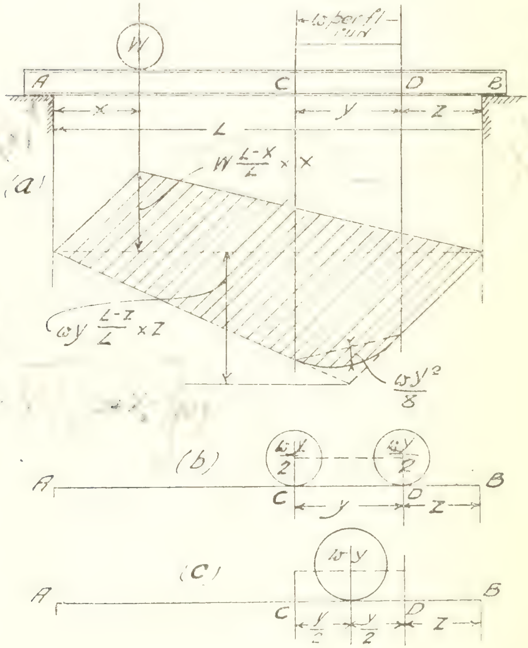


FIG. 54.

the line by the method described above. The ordinate indicating the B.M. at any point P can then be measured on both diagrams at once, as $p' p''$.

The case of a partly distributed and partly concentrated load may be treated in the way shown in Fig. 54. As the bending moments in A C and D B due to the distributed load depend entirely upon the reactions R_A and R_B these reactions would be the same if the load were (*a*) distributed as shown, (*b*) concentrated at C and D, or (*c*) concentrated at their centre of gravity, and therefore the bending moment diagrams for A C and D B are identical for all three cases. As far as these portions of the girder are concerned the load may therefore be treated as in *c* or with a chord cut off as in *b*. If on this chord an arc be superimposed with a central ordinate equal to the bending moment $\frac{wy^2}{8}$ on y treated as a little independent girder the diagram will be completed.

When it is necessary either in a girder or a cantilever to strengthen the flanges by riveting on addi-

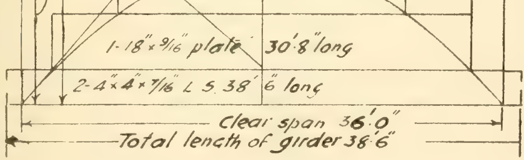

Flange area provided 21·4□"
Net " " required 21·1□"
18·0
15'4"
10'8"
1-18"×9/16" plate 21'4" long
1-18"×9/16" plate 30'8" long
2-4"×4"×7/16" L S 38'6" long
Clear span 36'0"
Total length of girder 38'6"

FIG. 55.

tional plates, the length of those plates is determined by means of a diagram of bending moments in the manner shown in Fig. 55, the cross-sectional area necessary dropping off with the decreasing bending moment which calls for that area.

When used for this purpose a flat parabolic curve demands very accurate drawing. The very simple "Pearce" method shown in Fig. 56 gives a curve as steep as necessary with correspondingly accurate results. It has the additional advantage that it can be drawn on ordinary foolscap with the rest of the calculations. The object being only to find points in a parabolic curve there is no necessity to draw the curve in at all, and it is usually only sketched in roughly.

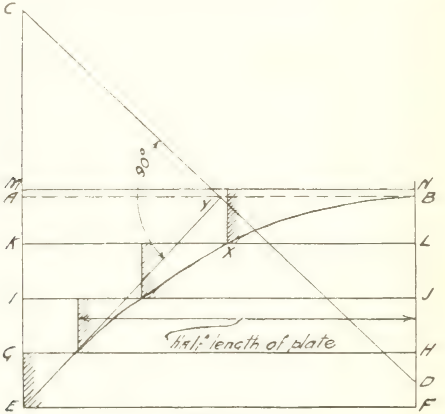


FIG. 56.

METHOD OF ASCERTAINING THE LENGTH OF PLATES IN GIRDER FLANGES BY FINDING POINTS

IN A PARABOLA.—Set out half clear span of girder to any convenient scale, as E F, Fig. 56, and set up the *necessary* sectional area of plates E A in the centre of the girder to a convenient scale, and the cross-sectional area of different plates decided upon, E G, G I, I K, K M, etc. Then to find any point as X in a parabola, cutting off lengths of plates, bisect A B at Y, join Y E and draw C D at 90° with E Y. With D as centre and radius C K (to bottom of plate required) strike off L X, equals half length of plate, X being a point in the parabolic curve E X B.

Care should always be taken that the length of plate specified will enable the ends to fall between two rivets.

For cases where a large number of calculations have to be typed in duplicate for presentation to a certifying authority, and diagrams have to be avoided as much as possible, the author has devised the following arithmetical method, but for general office work the method of Mr Pearce is quicker and more simple.

FORMULA FOR ASCERTAINING LENGTH OF PLATES IN GIRDER FLANGES.

Let A = necessary sectional area at centre.
a = sectional area of top plate included in parabola = A − area of all plates and angles except top plate.
a' = sectional area of second plate $+ a$.
a'' = ,, ,, third ,, $+ a' + a$.
a''' = ,, ,, fourth ,, $+ a'' + a' + a$, etc.
$L = \frac{1}{2}$ length of top plate,
$L' = \frac{1}{2}$,, second plate,
$L'' = \frac{1}{2}$,, third ,,
$L''' = \frac{1}{4}$,, fourth ,,
$K = \frac{1}{2}$ span.

$$L = \frac{K\sqrt{a}}{\sqrt{A}},\quad L' = \frac{K\sqrt{a'}}{\sqrt{A}},\quad L'' = \frac{K\sqrt{a''}}{\sqrt{A}},\quad L''' = \frac{K\sqrt{a'''}}{\sqrt{A}}, \text{ etc.}$$

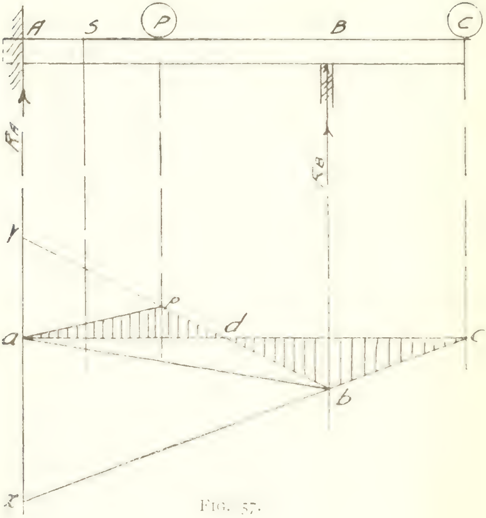


FIG. 57.

CALCULATION OF BENDING MOMENTS ON BALANCED CANTILEVERS.—A case which not infrequently arises in practice is that of a beam loaded between two supports, and projecting over one of the ends as in Fig. 57. To find the B.M. at any point S, either the reaction R_A or the reaction R_B must be first ascertained. The clockwise moments which determine R_B are C × C A and P × P A, the anticlockwise moment being $R_B \times A\,B$.

$$R_B \times AB = (C \times AC) + (P \times AP).$$

$$R_B = \frac{(C \times AC) + (P \times AP)}{AB}.$$

The B.M. at any point S would be

$$(R_B \times BS) - [(P \times PS) + (C \times CS)], \text{ or } R_A \times AS.$$

The reaction at A will be an upward or a downward force according as to whether the moment exercised by C, *i.e.*, C × C B, be greater or less than that exercised by P, *i.e.*, P × P B, or the two may just balance and relieve A of any reaction at all either upward or downward. The B.M. at S being R_A × A S it follows that at S there may be no bending moment due to P and C. The case may be solved graphically as shown. The ordinate *a x* is set up to represent the moment C × C A, *x c* is joined and the triangle of bending moments *a x c* due to the weight *c c* is formed. The contrary moment of the reaction at B, viz., B × A B, which is also represented by *a x*, is sheared off the last triangle, leaving the triangle *a b c*. If upon *a b*, as a new base line, be superimposed the triangle *a p b* to represent the moments on the span A B due to any load P, one complete diagram for the whole is obtained. Treating A B without regard to B C, *a v* is set up equal to the reaction at B (due to P) × A B, and from the triangle *a v b* is sliced off the triangle *a v p*, as before described.

Part of the bending moments in the span are, however, relieved by the weight C, and the extent of this relief and the effective B.M. which remains is shown by the shaded parts of the diagram.

It will be seen that from *a* to *d* the tendency of the bending moments is to cause the beam to sag or bend downwards, whilst from *d* to *c* the tendency is to cause hogging or bending upwards. From Fig. 58 it will be seen that the weight C may be so large in proportion to the weights in the span A B, that the net tendency will be to cause hogging tendencies in the whole length A B. On the other hand, the moment of the loads on A B may be just sufficient to balance C, leaving only hogging tendencies between P and C.

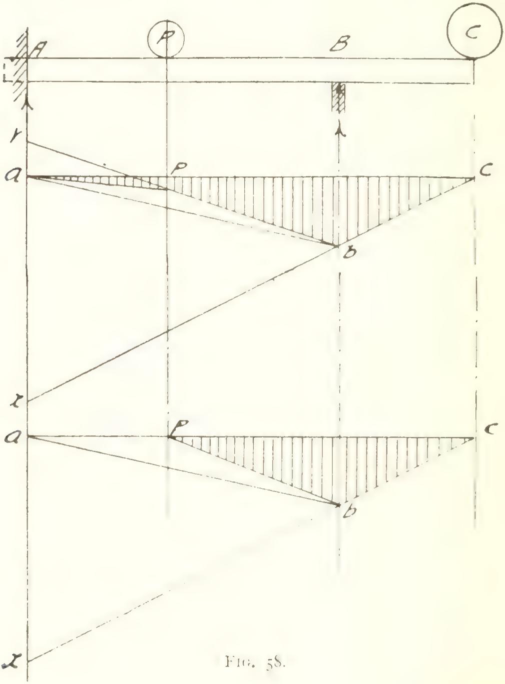


FIG. 58.

In Fig. 59 is shown the case of a rolled joist subject over part of its length to a slight bending moment, but over the central bearing B to a very severe one. The necessary flange area is supplied by extra plates whose lengths are determined by a diagram similar to those just described.

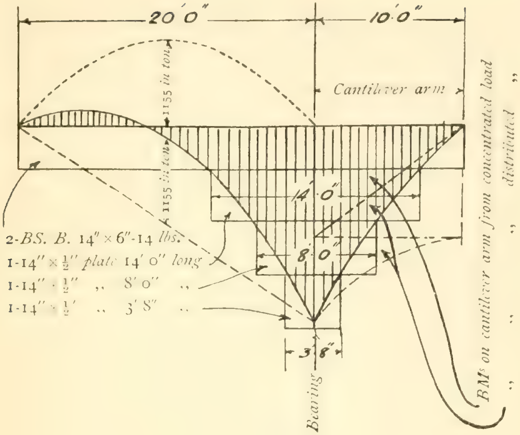


FIG. 59.

GIRDERS WITH FIXED ENDS AND CONTINUOUS GIRDERS.—Girders of a span L with rigidly fixed ends may be regarded as consisting of a span of $\frac{1}{2}$L supported on two cantilevers of a length each equal to $\frac{1}{4}$L. But girders with really fixed ends are in building practice extremely rare. Brickwork, stonework, and concrete are much less rigid than steel, and built-in girder ends cannot depend upon these materials for absolute rigidity. The limit of deflection which is generally fixed to prevent the cracking of plaster is $\frac{1}{40}$ in. per foot of span. A girder end built in so as to be really fixed would require to be constrained against adopting such a slope as would permit of this deflection. If the built-in portion were, say, 12 in. long, the extreme end would have to be prevented from rising, say,

$\frac{1}{20}$ in., otherwise the girder would be free to act as an independent span. A glance at a scale of $\frac{1}{8}$ in. to 1 ft. will show that $\frac{1}{20}$ in.—representing as it does about 4 in. to such a scale—is a distance so extremely small that it would be ridiculous to depend upon brickwork or masonry to prevent such an almost infinitesimal movement.

POINTS OF CONTRAFLEXURE.—In the same way a girder continuous over two or more spans, as in Fig. 60, might be expected to be much stronger than

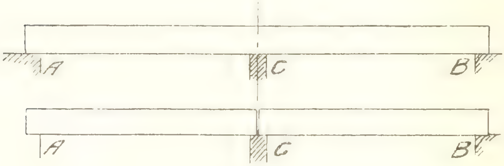
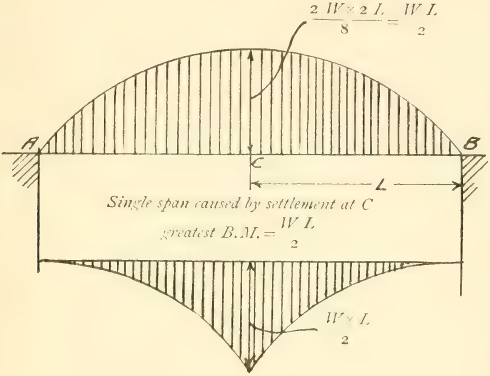


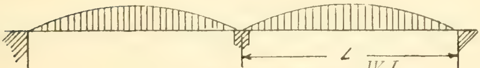
FIG. 60.

two independent spans, the bending changing from sagging to hogging at the points *c f*, Fig. 61, which are called the points of contrary flexure. But with continuous loading the B.M. over C would be as severe as in the centre of independent spans A C or C B, and the assumption of any relief afforded to these spans depends upon the three supports A, B, and C being built and also settling exactly in line. If C were slightly lower than a line joining A and B, the girder would act as one span A B, whilst if it were slightly higher it would act as two cantilevers C A and C B, both of which conditions are, as will be seen from the bending moment diagrams, distinctly less favourable than if the girder were divided at C into two independent spans. It is, therefore, becoming customary in good practice not only not to depend upon the continuity of girders, but also to deliberately sever them over the points of support. The theory of continuous

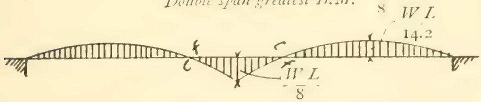


Single span caused by settlement at C
greatest B.M. = $\frac{WL}{2}$

Double cantilever caused by settlement at A or B
greatest B.M. $\frac{WL}{2}$



Double span greatest B.M. $\frac{WL}{8}$



Theoretical continuous span greatest B.M. $\frac{WL}{8}$

FIG. 61.

girders is both complicated and incomplete, but as the practical conditions of ordinary building almost invariably prevent the continuity from being depended upon, it is hardly necessary for the student to devote much time and labour to its investigation.

The *theoretical* assumption is that a fixed end to a span reduces the effective span L by about one quarter, the bending moments in a girder with two fixed ends and uniformly loaded being $\frac{WL}{24}$ at the centre and $\frac{WL}{12}$ at the bearings.

The method of calculation usually adopted is based upon that described by Professor Fidler in a paper on "Continuous Girder Bridges," *Trans. Inst. Civil Engineers*, vol. lxxiv.*

REACTION OF SUPPORTS OF CONTINUOUS BEAMS.—Although the rigidity of supports cannot be relied upon, its possible effects must not be ignored. The following table gives the theoretical reaction of the supports of continuous beams, the loading of a single bay being 1. It does *not* allow for faulty alignment or irregular settlement of supports, which, as will be seen from Fig. 61, may impose upon one support the weight of two or more bays:—

NUMBER AND REACTION OF SUPPORTS.

No. of Spans.	1	2	3	4	5
1	0·5	0·5	...	...	...
2	0·375	1·25	0·375	..	...
3	0·4	1·1	1·1	0·4	...
4	0.393	1·14	0·928	1·14	0·393

THE USE OF THE FUNICULAR POLYGON AS A DIAGRAM OF BENDING MOMENTS.—There is a striking similarity between the form assumed by a chain loaded with a series of weights, and a diagram of the bending moments which those weights would inflict upon a girder of the same span. One has an instinctive feeling that if the proper scale could be found the ordinates of such a funicular polygon would prove to be measures of the bending moments. This is actually the case, and it forms a very useful property of the funicular polygon. It can be proved as follows :—

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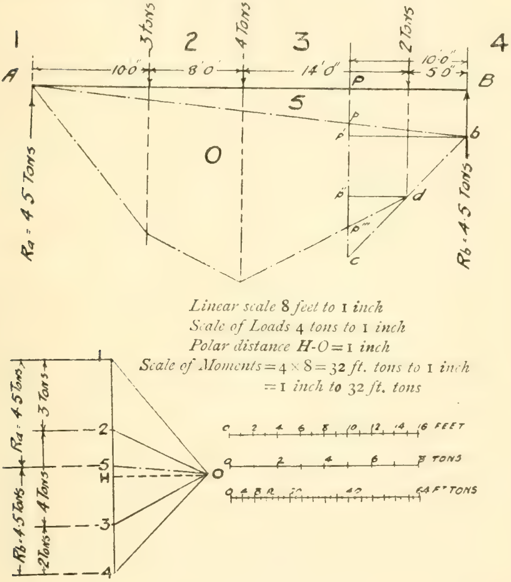
* See also "Bridge Construction" by the same author (Griffin).



Linear scale 8 *feet to* 1 *inch*
Scale of Loads 4 *tons to* 1 *inch*
Polar distance $H\text{-}O = 1$ *inch*
Scale of Moments $= 4 \times 8 = 32$ *ft. tons to* 1 *inch*
$=$ 1 *inch to* 32 *ft. tons*

FIG. 62.

In any girder, as A B, Fig. 62, the bending moment at any point is equal to the turning moment of the reaction at either bearing, less the contrary moment of any weights between the point and that bearing.

Thus the moment at P = (4.5 tons × 10 ft.) − (2 tons × 10 ft. − 5 ft.) = 35 ft.-tons. On the polar diagram, Fig. 62, the reaction R B = 5, 4.

The moment at P due to R b = 5, 4 × B P = 5, 4 × bp'.

Triangles pbc and 5, 0, 4 are similar.

$$\therefore \quad bp : pc :: 0, 5 : 5, 4.$$

Similarly, $\therefore \quad bp' : pc :: OH : 5, 4.$

$$\therefore \quad 5, 4 \times bp' = pc \times OH.$$

∴ The moment at P due to R b = pc on stress diagram × O H.

The moment at P due to the load 3, 4 = 3, 4 × dp''.

Triangles 4, 0, H, and $dp''c$ are similar.

$$\therefore \quad dp'' : p'''c :: OH : 3, 4.$$

$$\therefore \quad 3, 4 \times dp'' = p'''c \times OH.$$

∴ The moment at P due to the load 3, 4 = $p'''c$ on stress diagram × O H.

The net moment at P = (Rb × 10) − (2 × 5)

= (pc × OH) − ($p'''c$ × OH)
= pp''' (the ordinate of the stress diagram under point p) × OH.

The length of any ordinate × O H is, therefore, a measure of the bending moment over that ordinate; or the length of the ordinate is proportional to the bending moment divided by O H. The length of any ordinate such as pp''', and, therefore, the scale of foot-tons, is determined by—

1. The scale to which the girder is drawn, or scale of feet.

2. The scale to which the loads are drawn, or scale of tons.

3. The polar distance O H.

The larger the linear scale to which the girder is drawn, the larger will be the ordinates in the funicular polygon and the fewer will be the number of inch-tons that any given length of ordinate will represent to the scale of foot-tons required. In other words, the fewer the number of feet represented by 1 in. on the linear scale, the fewer will be the number of foot-tons represented by 1 in. on the scale of bending moments. Similarly, the larger the scale of tons to which the loads are drawn the larger will be the ordinates and the scale of foot-tons, *i.e.*, the fewer the number of tons represented by 1 in. in the scale of tons the fewer will be the number of foot-tons represented by 1 in. in the scale of bending moments. The longer O H is drawn the shorter the ordinates will be, *i.e.*, the greater the number of inches in O H the greater will be the number of foot-tons represented by 1 in. in the scale of bending moments.

If the linear scale is x ft. to 1 in. and the force scale is y tons to 1 in., then the number of foot-tons represented by 1 in. on the scale of bending moments would be $\frac{x \times y}{\text{OH}}$. If the girder be drawn to a scale of $\frac{1}{2}$ in. to 1 ft. (2 ft. to 1 in.) and the loads to a scale of $\frac{1}{2}$ in. to 1 ton and O H be 4 in., then the number of foot-tons represented by 1 in. on the scale of bending moments would be $\frac{2 \times 2}{4} = 1$ ft.-ton to 1 in. If the linear scale were $\frac{1}{8}$ in. to 1 ft., the scale of tons $\frac{1}{10}$ in. to 1 ton, and O H 2 in., then the scale of bending moments would be $\frac{8 \times 10}{2} = 40$ ft.-tons to 1 in.

CHAPTER XII

MOMENT OF RESISTANCE

The Calculation of Symmetrical Beam Sections—Varying Efficiency of Beam Fibres—Section Modulus (z)—z of Rectangular Sections—Modulus of Rupture—Relation between Section Modulus and Strength of a Rolled Joist—Calculation of Beam Sections in a Small Footbridge—Selection of Sections not Determined by Considerations of Strength Only—Handy Formulæ for Beams—The Calculation of z of Unsymmetrical Sections—Moment of Inertia—Inertia Areas—Relation of Moment of Inertia to Section Modulus—Computation of Moment of Inertia—Numerical Coefficient of Moment of Inertia of a Section—Infinite Number of Moments of Inertia in any Section—Section Modulus of Unsymmetrical Sections—Calculation of Brick and Masonry Buttresses — Deduction of Strength of Joists under Irregular Loadings from Strength under Distributed Load.

THE CALCULATION OF SYMMETRICAL BEAM SECTIONS.—Having considered the forces attacking a beam or cantilever and calculated their fighting values under the term of Bending Moment, the next step is to compute in similar terms the powers of defence possessed by the beam, which are summed up in the term Moment of Resistance.

The external loads on a beam, acting at various advantages of leverage, tend to tear the lower parts of the beam asunder and to crumple the upper parts. The fibres of the beam will resist these tendencies, and will hold themselves together against pairs of tensional forces and push out against pairs of compressive forces. The direction of their effort is always opposite to that of the attacking forces, and they act with whatever advantage of position or leverage that may be given to them by the shape of the cross section of the beam. It should be carefully borne in mind that a beam never exerts a moment of resistance greater than is necessary to resist the attack made upon it. A railway bridge, by virtue of the depth of its girders, the area of their flanges and the latent power of those steel muscles, may be capable of exerting, *if necessary*, a moment of resistance equal to the huge bending moment caused by some hundreds of tons of train load ; but it does not make that effort every time a platelayer walks over—it only exerts a moment of resistance exactly equal to the bending moment caused by the weight of the platelayer.

It is necessary to keep very clearly in mind the apparently obvious fact that action and reaction, bending moment and moment of resistance, are not only equal and opposite, but *distinct.* Bending moment is compounded of external loads and reactions acting with certain advantages of position or leverage, and their net tendency is to distort any beam or structure which may be opposed to them, irrespective of whether that beam be deep or shallow, strong or weak, or composed of steel or paper. The attack is the same whatever may be the strength or weakness of the defence. The inherent power of resistance which a beam opposed to bending moment may possess, depends upon the strength of its fibres, their size, and their disposition in the cross section—it has nothing whatever to do with the span or load or any other constituent of the *bending moment.* The only connection between the two is the important one that they are always exactly equal to each other, and it is as well to commence every calculation in connection with beams and cantilevers with the simple equation B.M. = M.R., and to deduce from that any unknown factors. It is necessary to be particularly careful in order to avoid the very tempting error of writing length (L) in feet on one side of the equation, and depth (*d*) or breadth (*b*) in inches on the other side. All factors in an equation must be reduced to the same denomination, and if *d* is taken in inches, *b* and L must be taken in inches.

VARYING EFFICIENCY OF BEAM FIBRES.—It is fairly obvious that in all girders or cantilevers deflected under load the outer layers of fibres are being punished more severely than those nearer to the centre. This can be very clearly seen if a number of wires be loosely threaded through holes pierced longitudinally in a model wooden rectangular beam, as in Fig. 63, the protruding ends being all of the

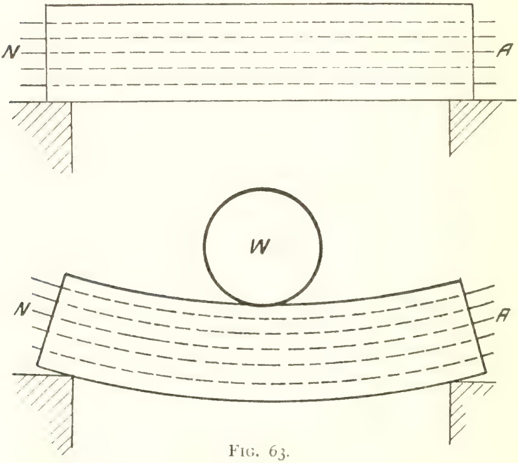


FIG. 63.

same length. When such a beam is deflected under a load, the fibres surrounding the holes are compressed in the layers above, and extended in those below the centre; but the wires themselves, being loose in their holes, are unaffected; and the alterations which take place in the lengths of their protruding ends, when carefully measured, show clearly the following important points :—

1. That there is one layer of fibres which is neither extended nor compressed, and that this layer, termed the neutral axis (N.A.), is always situated at the centre of gravity of the cross section.

2. That the remaining layers are extended or compressed unequally.

3. That the extent to which they are so strained is proportional to their distance above or below the neutral axis.

Every layer in the cross section above or below the neutral axis, however narrow that layer may be, is doing *something*, and every part of the web of a rolled joist or a thin-webbed plate girder, as Figs. 64 and 65, is adding its quota, however small, to the general moment of resistance of the

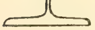


FIG. 64.

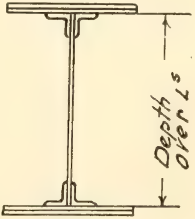


FIG. 65.

cross section. But it is very much easier to trace the action of a typical moment of resistance if at first the assumption is made that such a thin web takes no part in the moment of resistance, and that the flanges are acting at their centres of gravity. For reasons which will be shown later, neither of these assumptions are *quite* correct. They are very nearly so in thin-webbed beams, but if assumed in other sections they might lead to serious errors. The loaded beam can be considered in its simple form of a see-saw as consisting essentially of two double-headed bell-crank levers as in Fig. 66. The tendency of the weight W is to rotate one-half of the beam in an anti-clockwise direction round the neutral axis at C, and the tendency of the weight W′ is to rotate the other half in a clockwise direction

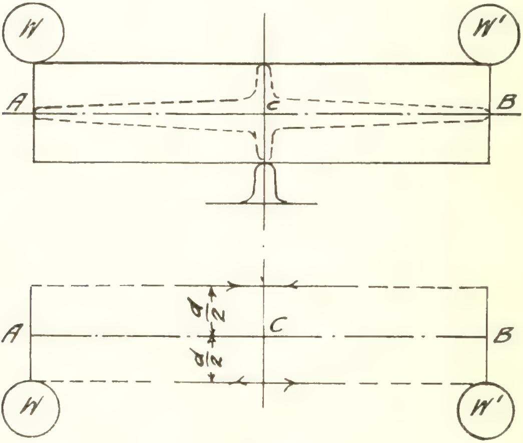


Fig. 66.

round the same point—the *two* tendencies forming *one* bending moment at C equal to W × A C *or* W′ × B C. This bending moment is resisted by the capacity of the particles of the top flange to hold themselves together in opposition to a pair of tensional forces, and by the capacity of the particles of the lower flange to push themselves apart in opposition to a pair of compressive forces; the *amount* of that capacity in each case being the net area (a) of the flange × the strength (f) of the material per square inch in compression or tension. The *leverage* at which the capacity of each flange acts is *very nearly* the distance $\frac{d}{2}$ between the centre of gravity of the cross section of the flange and the neutral axis. The action of these resisting flange stresses will be clear if they be considered as clips capable of resisting compression or tension, and connecting the top and bottom of the short arms of the bell-crank levers. The total moment of resistance (M.R.) is the pair of anti-compressive forces $\times \frac{d}{2}$ plus the pair of anti-tensional forces $\times \frac{d}{2}$. When the neutral axis is midway between the flanges (as it is in all ordinary symmetrical steel beam sections) these pairs of forces are equal, and the M.R. may be represented as *one* pair of forces × d. In other words, the area (a) of one flange × the permissible stress (f) per square inch × d or M.R. $= a\,d\,f$.

The moment of resistance, therefore, consists of at least three factors, the flange area (a), the effective depth (d) between the flanges, and the permissible flange stress (f). The permissible flange stress is the ultimate strength divided by the factor of safety, say 7 or $7\frac{1}{2}$ tons per square inch for mild steel. The factors a and d are permanently fixed when the girder is made and the *actual* stress in the fibres varies with the B.M. inflected on the beam, thus :—

$$\text{B.M.} = \text{M.R.} = adf.$$

$$f = \frac{\text{BM}}{ad}.$$

SECTION MODULUS (Z).—The part of the M.R. represented by ad is called the section modulus (Z). It is purely arithmetical and is the same for any particular profile of section whether it be of steel or soap, although the permissible stress f of course depends upon the material. Being independent of span, load, or material, the section modulus can be calculated for any given profile of section, and its exact value is tabulated for all British Standard and most foreign rolled sections. For plate and lattice girders it is approximately calculated in practical design by neglecting the web, and making the safe assumption that the flange areas are concentrated at each end of the "depth over angles" d in Fig. 65.

For rolled joists it can be approximated by neglecting the web and treating the flanges as if they were thin laminæ equidistant from the neutral axis. Thus in a 12 × 6 **I** section with flanges 6 in. × 0.72 in. (Fig. 71, page 150), the two flanges might be treated as if they were actually infinitely thin laminæ, each of 4.32 sq. in. in area and situated 5.64 in. from the neutral axis. The Z of the section would then be approximately $4.32 \times 11.28 = 49$, *i.e.*, the area (a) of one flange by the effective depth (d) between the centres of flanges, or—

$$Z = ad.$$

The actual Z of a standard 12 × 6 rolled joist, including the strength of the web, is 52.573, an error of about 7 per cent. on the safe side.

Z OF RECTANGULAR SECTIONS.—For rectangular sections, such as wood beams which have neither defined webs nor flanges, Z can be accurately arrived at by assuming hypothetical flanges as follows. The experiment with threaded wires proves conclusively that the strain or distortion in each thin layer of fibres decreases regularly from the outer edges to the neutral axis in a manner which would be represented by the horizontal ordinates in the two triangles at *a* in Fig. 67. The same ordinates would correctly represent the stress, because within the elastic limit strain is an exact measure of the stress which causes it. If instead of being rectangular, the two halves of the beam were triangular in section, as at *b*, the stress intensity per square inch would be the same in every layer because, as the stress increases above or below the neutral axis from layer to layer, the areas of the layers called upon to provide that stress would increase in the same proportion. Rectangular sections are therefore assumed to act as if they consisted of two triangles joined together at

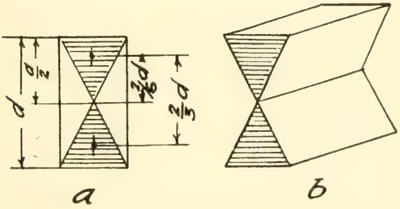


FIG. 67.

their apexes, and in consequence equally stressed in every fibre. The area (a) of each hypothetical triangular flange would be $\frac{bd}{2 \times 2}$, and would exercise the same moment as it would do if it were concentrated at its centre of gravity, as proved by the brick experiment described in Chapter V.

The centre of gravity of every triangle being at one-third the altitude, the two triangular flange areas would act as if concentrated in thin layers, each situated at a distance of one-third of the height $\left(\frac{d}{2}\right)$ of each triangle, or $\frac{2}{3}d$ apart.

The expression $a\,d$ for the section modulus Z of any rectangular section would therefore read—

$$\text{area of one triangular flange} \times \frac{2}{3}d = \frac{bd}{4} \times \frac{2}{3}d = \frac{bd^2}{6}.$$

This expression $\frac{bd^2}{6}$ should be carefully committed to memory. It is not only applicable to wooden beams and cantilevers, but is used in all cases where a rectangular section is subjected to bending or overturning moment, as in the design of walls, foundations, buttresses, arches, stanchions, retaining walls, &c.

MODULUS OF RUPTURE.—Although in thus considering the M.R. of a rectangular section no such incorrect assumption is made as was done in the case of the rolled beam, yet it should be noted that all sections which have a large excess of web will stand a higher stress than can be satisfactorily accounted for when compared with the results obtained by testing specimens of the material to destruction by direct tension or compression. This higher stress is termed the Modulus of Rupture (*fo*) of the material. Fortunately it is practically the same whether the section be large or small; so that an *fo* deduced from the breaking of a small stick can be applied to a large girder. For instance, if it were desired to ascertain the safe distributed load, using a factor of safety of 9, on a 6-in. × 12-in. beam, 15 ft. span, of some foreign wood, of which three sticks, 1 in. × 2 in. × 3 ft. long, broke under an average central load of 6 cwt., the necessary calculation would be as follows:—

Deduction of fo *from breaking of test sticks*—

$$\text{B.M.} = \text{M.R.}$$

[illegible] at centre

$$\frac{\text{WL}}{4} = \frac{bd^2 fo}{6},$$

$$\therefore \quad fo = \frac{\text{WL} \times 6}{4 \times bd^2} = \frac{6 \times 36 \times 6}{4 \times 1 \times 2 \times 2} = 81 \text{ cwt. per square inch.}$$

Permissible stress—

$$f = \frac{fo}{\text{Factor of safety}} = \frac{81}{9} = 9 \text{ cwt. per square inch.}$$

Safe distributed load on 12-*in.* × 6-*in. beam*, 15 *ft. span, then* f = 9 *cwt.*

$$\text{B.M.} = \text{M.R.}$$

[illegible]

$$\frac{\text{WL}}{8} = \frac{bd^2 f}{6}.$$

$$\text{W} = \frac{bd^2 f \times 8}{6 \times \text{L}} = \frac{6 \times 12 \times 12 \times 9 \times 8}{6 \times 15 \times 12} = 57.6 \text{ cwt.}$$

In thin-webbed sections, such as rolled joists and plate and lattice girders, which are calculated without, or practically without, any such incorrect assumption, this anomaly disappears, and *f* can be taken as a proportion of the ultimate strength of the material as determined by the strength of test pieces in direct tension or compression. If, for instance, British Standard structural steel is manufactured to break at 26 to 32 tons per square inch, a factor of safety of 4 would limit the permissible working stress to, say, $7\frac{1}{2}$ tons per square inch, and the greatest moment of resistance in inch-tons for which any section could be called upon, *i.e.*, the greatest bending moment in inch-tons which it could safely stand, would be its $Z \times 7\frac{1}{2}$.

Relation Between the Section Modulus and the Strength of a Rolled Joist.—A 6-in. × 5-in. – 25 lbs. Standard rolled joist, for example, has, according to the Standard tables, a Z of 14.55 in the direction of its web and 3.64 in the direction of its flanges. It could therefore be used to withstand a bending moment of 14.55 × 7.5 in.-tons if fixed in the usual way, and 3.64 × 7.5 in.-tons when laid on its side.

Calculation of Beam Sections in a Small Footbridge.—To take another example—it is proposed to form a footbridge, 20 ft. span and 4 ft. 6 in. wide, of two rolled joists with cross bearers of **T** steels or small rolled joists laid across, 3 ft. apart, carrying stone slabs, the live and dead loads on the bridge being equivalent to $2\frac{1}{2}$ cwt. per foot super. dead (Fig. 68).

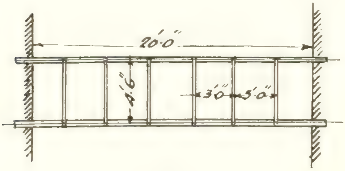


Fig. 68.

Calculation of Section Modulus Required in Cross Bearers.—

$$\text{B.M.} = \text{M.R.}$$

$$\frac{WL}{8} = Zf.$$

$$W = \frac{3 \times 4.5 \times 2\frac{1}{2}\ \text{cwt.}}{20} = 1.68\ \text{tons.} \quad L = 54\ \text{in.} \quad f = 7.5\ \text{tons.}$$

$$\frac{WL}{8} = \frac{1.68 \times 54}{8} = 11.37\ \text{in.-tons.}$$

The section to be provided must, therefore, possess a Z which, when multiplied by $7\frac{1}{2}$ tons, will at least equal 11.37.

$$11.37 = Z \times 7.5.$$

$$Z = \frac{11.37}{7.5} = 1.52.$$

On referring to the tables it will be found that a 3-in. × 4-in. × $\frac{7}{16}$-in. – 9.77 lbs. T.S. with a Z in the direction of the web of 1.62 is the lightest **T** section which could be used, whereas the substitution of a 4-in. × $1\frac{3}{4}$-in. – 5 lbs. **I** section with a Z of 1.83 would effect a gain of over 10 per cent. in strength with a saving of nearly 50 per cent. in weight.

The B.M. in the main girders would be—

$$\text{B.M.} = \text{M.R.}$$

$$\frac{WL}{8} = Zf.$$

$$W = \frac{20 \times 4.5 \times 2.5}{2 \times 20} = 5.63\ \text{tons.}$$

$$L = 240\ \text{in.}$$

$$Z = \frac{WL}{8f} = \frac{5.63 \times 240}{8 \times 7.5} = 22.5.$$

A 10-in. × 5-in. – 30 lbs. R.S.J. with a Z of 29.14 would be strong enough, but a 12-in. × 5-in. – 32 lbs.

with a Z of 36.69 would probably be used to avoid undue deflection.

With such a large excess of strength it is obvious that the weight of the joist itself would be easily carried, but the habit of neglecting dead weight calculations when they are apparently unnecessary is similar to the habit of slack sentry duty in peace time and has similar results.

Greatest Working Stress on 12-*in.* × 5-*in. R.S.J.*—

Weight of R.S.J. $= \dfrac{20 \times 32}{2240} = 0.286$ ton.

" bearers $= \dfrac{4.5 \times 5 \times 6}{2240} = 0.0602$ "

Total weight of R.S.J. and bearers $= 0.346$ ton.

Weight of floor load $= 5.630$ "

5.976, say 6 tons.

$$\text{B.M.} = \text{M.R.}$$

$$\frac{WL}{8} = Zf.$$

$$f = \frac{WL}{8Z} = \frac{6 \times 240}{8 \times 36.69} = 5 \text{ tons per square inch.}$$

SELECTION OF SECTIONS NOT DETERMINED BY CONSIDERATIONS OF STRENGTH ONLY.—After the designer has determined the strongest and most economical sections, he must ascertain whether they can be obtained and transported without difficulty. Then he must also consider whether the situation is one where exceptional corrosion or want of periodical painting does not render advisable the use of heavier sections than are necessitated by considerations of strength alone.

HANDY FORMULÆ FOR BEAMS.—It is an excellent rule, both in study and in practical design, to always keep quite distinct the constituent terms of B.M. and M.R., and to commence working out every problem by placing them in true opposition; but it is worth while straining this rule a little in order to obtain short handy formulæ, easily remembered, which can be used when time is short and when data of section moduli are not available. The two following formulæ give the safe distributed loads on wood joists and steel beams. All the dimensions are used in their most natural form, viz., those relating to the cross section are taken in inches, the span in feet, the loads on wood in cwt., and the loads on steel in tons. In this case the span or length in feet is written as S instead of L to conform with the notation of most text-books in which these formulæ appear. The permissible stresses are the safe ones of 9 cwt. per square inch on wood and 6 tons on steel. In the case of wood beams—

$$l \text{ in } \textit{inches} = \text{span S in } \textit{feet} \times 12.$$

$$\text{B.M.} = \text{M.R.}$$

$$\frac{Wl}{8} = \frac{bd^2f}{6}.$$

$$W = \frac{bd^2 9 \times 8}{6 \times 12 \times S} = \frac{bd^2}{S}.$$

∴ Safe distributed load on wood beams in cwt.

$$= \frac{bd^2 \text{ in inches}}{S \text{ in feet.}}$$

In the case of steel girders—

$$l \text{ in inches as before} = S \text{ in feet} \times 12.$$

$$a = \text{area of one flange in square inches.}$$

$$\text{B.M.} = \text{M.R.}$$

$$\frac{Wl}{8} = adf.$$

$$W = \frac{ad \times 6 \times 8}{12S} = \frac{4ad}{S}.$$

∴ Safe distributed load on steel girders in tons

$$= \frac{4ad \text{ in inches}}{\text{S in feet.}}$$

The safe central concentrated load would be one-half the above values.

THE CALCULATION OF Z OF UNSYMMETRICAL AND IRREGULAR SECTIONS.—Beam sections in common use are confined to rectangular wooden beams, rolled sections, and plate or lattice girders. For the practical design of ordinary beams in which the position of the flanges can be assumed with sufficient accuracy, or of which the section modulus can be obtained from standard tables, the foregoing methods will suffice. But moments of resistance have also to be calculated for many other sections—for arches, for buttresses irregular on plan, for **T** and **L** bars used as beams, and for struts with sections of infinite variety, in which it is not possible to make the same simple approximations which can be made in the case of ordinary beams, and which make the calculations of the latter very easy to follow. In order, therefore, to be in a position to calculate the strength of every possible section, it is necessary to go through the subject again rather more rigorously, avoiding those assumptions which are made in order to keep clear the main features of moment of resistance.

MOMENT OF INERTIA.—Experiments on a beam with threaded wires (Fig. 63) proved that the stress in any layer of a beam under load is proportional to its distance from the neutral axis. If the stress caused by any system of loading in a layer at 1 in. from the neutral axis be, say, 1 cwt. per square inch, then the stress at any outer layer can be deduced from it. At 3 in. from the neutral axis it would be

3 cwt. per square inch, at 5 in. from the neutral axis it would be 5 cwt., and so on. Conversely if the stress (f) in a layer of outer fibres at 5 in. from the neutral axis be known, then the stress at 1 in. from the neutral axis would be $\frac{f}{5}$.

But the leverage at which the stress in any layer works to resist bending moment is also its distance from the neutral axis; and the effective strength of that layer to resist bending moment is therefore determined by its area multiplied by the *square* of its distance from the neutral axis.

The total effective effort which a beam opposes to bending moment caused by any load (*i.e.*, its moment of resistance, M.R.), expressed in, say, inch-cwts., can be computed by ascertaining the stress in cwts. per square inch in a layer situated at a unit distance of 1 in. from the neutral axis (and therefore working at a unit leverage of 1 in.) by deducing from that the enhanced effort of every other layer further away from the neutral axis; and then adding together the different contributions to the total moment of resistance thus made by each layer.

The different layers considered must of course be infinitely thin if the calculation is to be mathematically correct, because the top and bottom laminæ of any layer of appreciable thickness would have different values, and the central lamina would have a value which

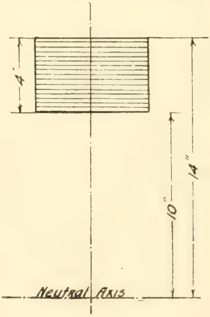


FIG. 69.

would *not* be the mean between that of the top and bottom.

For instance, in a layer (Fig. 69) 4 in. thick, the value of the top lamina would be its area (a) $\times$ 14^2 $= 196a$. The value of the bottom lamina would be similarly $a \times 10^2 = 100a$. The mean of these two values is $148a$, whereas the value of the middle layer is $a \times 12^2$ or $144a$.

It is this fact which renders incorrect the assumption that a flange acts at its centre of gravity. That assumption would be quite correct if the flange were equally stressed in every layer, which is *very nearly* the case in rolled joists, plate, and lattice girders.

INERTIA AREAS.—In rectangular sections a hypothetical triangular section of flange equally stressed in every layer is calculated, and in all sections it is possible to ascertain a similar hypothetical flange area generally termed the "inertia area"; but the experience of the author is that, except in the case of rectangular sections, the calculation of inertia areas saves neither time nor trouble.

To avoid an infinity of labour in calculating a great number of extremely thin laminæ, it is necessary to employ an arithmetical short cut provided by higher mathematics, viz., to multiply the stresses at 1 in. from the neutral axis by *an arithmetical expression which correctly indicates the sum of the areas of each infinitely thin layer in the section, multiplied by the square of its distance from the neutral axis.*

Such an arithmetical expression is called the Moment of Inertia (I) of the section—a term borrowed from the science of rotating bodies in which the inertia or resistance to moving of a revolving body such as a fly-wheel depends upon the size of each particle (viz., its area in cross section) multiplied by the square of its distance from the centre of rotation; the combined sum of all the particles multiplied by the square of their leverages expressing the "moment of inertia" of the whole wheel.

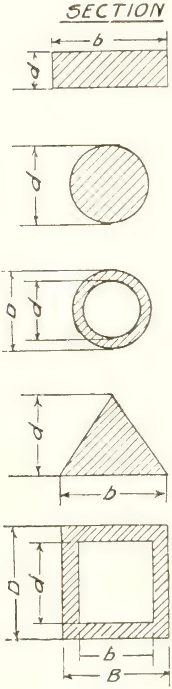
It will be observed that the term "moment of inertia" is an arithmetical one pure and simple, which relates *only* to the shape of the cross section, and, like the section modulus Z, is affected by no other consideration whatever. It is the same for a beam of steel 20 ft. long as for a bar of soap 12 in. long if they are the same in cross section. It can only be used to represent the M.R. which a section will oppose to any given B.M. if the stress per square inch at 1 in. from the neutral axis due to that B.M. be known.

This is arrived at as follows:—

RELATION OF MOMENT OF INERTIA TO SECTION MODULUS.—If the stress per square inch in the outermost layer of a beam under a given B.M. be called f, and its distance in inches above or below the neutral axis be called y, then the stress at 1 in. from the neutral axis would be $\frac{f}{y}$. The total M.R. would then be $\mathrm{I} \times \frac{f}{y}$ or $\frac{\mathrm{I}}{y} \times f$. But, as was shown previously, M.R. $= \mathrm{Z}f$, *and* $\frac{\mathrm{I}}{y}$ *is therefore simply an exact expression of the section modulus* Z. Like I, $\frac{\mathrm{I}}{y}$ or Z is a purely arithmetical expression of the *size* of the beam's muscles multiplied by their leverage. They may be strong or weak—hard or flabby; Z merely tells us their *size*, not their *strength*. Only in conjunction with f does it determine what muscular effort they are capable of—

$$\text{BM} = \frac{\text{I}}{y} f = \text{Z}f.$$

If f be the extreme intensity of muscular stress which the fibres of the beam can stand without



Moment of Inertia = I.

$$\frac{bd^3}{12}.$$

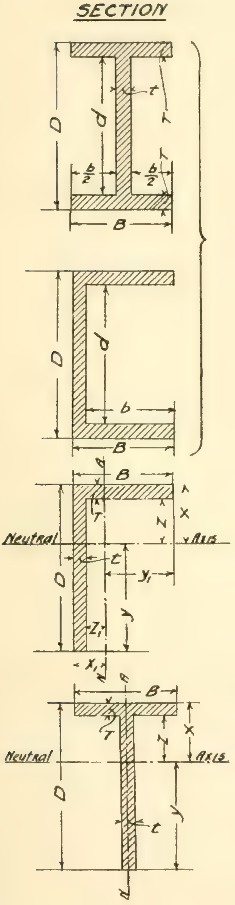
$$\frac{\pi d^4}{64} = 0.0491 d^4.$$

$$\frac{\pi(\text{D}^4 - d^4)}{64} = 0.0491 \text{D}^4 - d^4.$$

$$\frac{bd^3}{36}.$$

$$\text{Maximum} = \frac{\text{BD}^3 - bd^3}{12}.$$

FIG. 70[1].



Moment of Inertia = I.

Maximum = $\frac{BD^3 - bd^3}{12}$.

Minimum = $\frac{(2T \times B^3) + (d \times t^3)}{12}$.

Maximum = $\frac{B(x^3 - z^3) + t(y^3 + z^3)}{3}$.

Minimum = $\frac{D(x_1^3 - z_1^3) + T(y_1^3 + z_1^3)}{3}$

Maximum = $\frac{B(x^3 - z^3) + t(y^3 + z^3)}{3}$.

Minimum = $\frac{TB^3 + (D - T)t^3}{12}$.

FIG. 70^2.

injury, then $z f$ represents the total effort which the beam can be safely trusted to exert without hurting itself. Conversely, if z be the muscular development of the beam, and B.M. the bending moment to which it is subjected, then $\frac{\text{BM}}{Z} = f$ will be the stress thereby inflected on the outermost fibres.

COMPUTATION OF MOMENT OF INERTIA.—The value of I for ordinary profiles is shown in Figs. 70[1] and 70[2]. In rectangular sections the distance y is $\frac{d}{2}$ and the expression $\frac{\text{I}}{y}$ or $\text{Z} = \frac{bd^3}{12} \div \frac{d}{2} = \frac{bd^2}{6}$, the same result that was obtained by assuming a hypothetical equivalent triangular flange area equally stressed in every layer.

To obtain the moment of inertia of any section the arithmetical rule is—

a. Divide the total section into separate figures of definite shape, such as rectangles and triangles.

b. Ascertain the position of the centre of gravity of the whole section.

c. Calculate the moment of inertia of each figure.

d. Multiply the area of each figure by the square of the distance between the neutral axis of the whole section and a line drawn through the centre of gravity of the figure parallel to the neutral axis.

The sum of *c* and *d* will represent the moment of inertia of the section, and this divided by the distance of each outside layer (or any other layer) from the neutral axis, gives the section modulus Z *for that layer* and enables the stress in it to be calculated from $f = \frac{\text{BM}}{Z}$. The strength of a section being determined by the strength of its most heavily stressed part, z is calculated for the extreme outer layers of fibres.

CALCULATION OF MOMENTS OF INERTIA BY NUMERICAL COEFFICIENT.—The moments of inertia of sections about their neutral axes can generally be expressed in the simple form of $I = NAd^2$, in which A is the area of the section, d the total depth at right angles to the neutral axis, and N is a numerical coefficient depending upon the form of the cross section.

Thus for all square or rectangular sections $I = \frac{bd^3}{12}$; as the area $A = bd$, I can be correctly expressed for all such sections as $\frac{1}{12} Ad^2$, or NAd^2 if $N = \frac{1}{12}$. Similarly for all circular sections $I = \frac{\pi d^4}{64}$ and $A = \frac{\pi d^2}{4}$ and $I = \frac{1}{16} Ad^2$, or NAd^2 if $N = \frac{1}{16}$.

For annular sections or rings the exact expression of n in terms of d^2 is impossible, but the value $\frac{1}{8}$ is sufficiently correct for all practical purposes in the calculation of hollow columns or of the vertical rods of reinforced concrete columns when arranged in a circle, the coefficient reading $I = NAd^2$, in which $N = \frac{1}{8}$.

Similarly for hollow rectangles the coefficient is very nearly $\frac{1}{6}$, or $I = NAd^2$, in which $N = \frac{1}{6}$.

The use of these coefficients will be found to greatly reduce the labour of calculating column sections, especially those in reinforced concrete.

INFINITE NUMBER OF MOMENTS OF INERTIA IN ANY SECTION.—It is obvious that, as beams can be bent in any direction, there must be an infinite number of possible neutral axes at right angles to these directions. I should, therefore, always be designated with reference to the neutral axis about which it is calculated. Thus in the rolled beam shown in Fig. 71, the maximum I would be calculated round *x x* and be designated as the I for axis *x x*, and the minimum would be similarly designated as the I for axis *y y*.

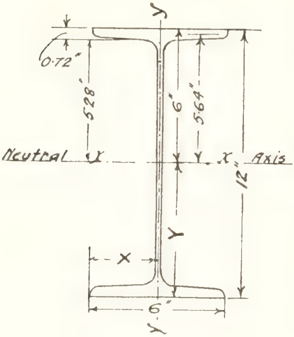


Fig. 71.

If the I for axis *x x* be 315.44, and the distance *y* 6 in., then the greatest modulus of the section would be—

$$\frac{\mathrm{I}xx}{6} = \frac{315{\cdot}44}{6} = 52{\cdot}573.$$

The least section modulus would be similarly—

$$\frac{\mathrm{I}yy}{3} = \frac{22.257}{3} = 7{\cdot}419.$$

Without exceeding a fibre stress of $7\frac{1}{2}$ tons per square inch, such a joist could be safely trusted to resist a bending moment of $52.573 \times 7\frac{1}{2}$ in.-tons when fixed in the ordinary way, and $7.419 \times 7\frac{1}{2}$ in.-tons if laid on its side.

Section Modulus of Unsymmetrical Sections.—It will also be obvious that in any section whose neutral axis is nearer to one edge than it is to the other, as in the **T** section, Fig. 72, the distance *y* of the extreme fibre from the neutral axis has two values according to whether the upper or the lower extreme fibre is being considered. Such a section, like every other section, has only one I about the neutral axis, but it has two obvious values for the section modulus, and when under load the expression $\frac{BM}{Z}$, which gives the extreme stress in the outer fibre,

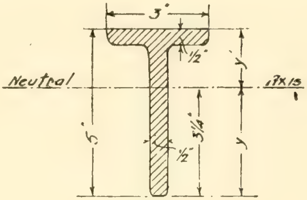


FIG. 72.

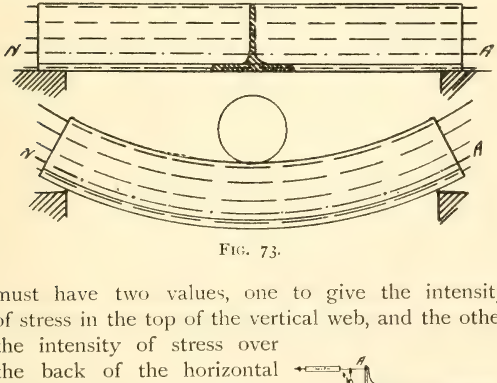


FIG. 73.

must have two values, one to give the intensity of stress in the top of the vertical web, and the other the intensity of stress over the back of the horizontal table. If such a T section be considered as deflected under a load, as in Fig. 73, it would be evident that the outer edge of the web is

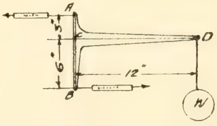


FIG. 74.

being punished to a greater intensity than the table. This is only what would be expected from the experiment of the beam with threaded wires; but it can be proved beyond question. If a double-headed bell-crank lever A B C D, Fig. 74, with unequal short arms A C and B C, be centred at C, and secured by spring balances of equal stiffness. Any downward pull at D will cause a pull to be registered at B greater than that at A.

Calculation of I of Brick and Masonry Buttresses.—It is generally found necessary to calculate a moment of inertia when dealing with **T**-shaped sections in brickwork and masonry, such for instance as the case (frequently met with in practice) of a buttress projecting from a wall, and subject to lateral thrust from the wind pressure on a steep roof (Fig. 75). The tendency of the wind pressure is to set up tension in the back of the wall, and the proportions of the but-

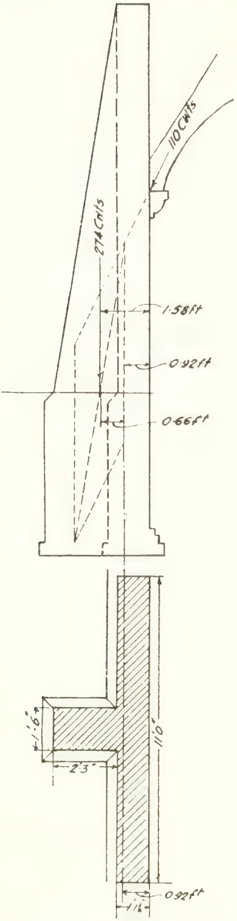


Fig. 75.

tress are determined with a view to neutralising this tendency. If the buttress be considered as acting alone, the sufficiency of any assumed section can be readily ascertained by compounding its weight with the lateral thrust, and ascertaining that the resultant of the two falls within the "middle third" (see Chapter XIX.). But the buttress is *not* acting alone—it cannot move without pulling down the wall to which it is bonded. What is really resisting the overturning moment of the force is a T section standing on end, of which the table represents the wall, and the web represents the buttress. At any section it is subjected to an initial intensity of compression $\frac{W}{A}$, A being the area of the section, and W the vertical component of the resultant in that section. In addition it is subjected to a bending moment from the lateral force tending to increase $\frac{W}{A}$ on one side, and to decrease it on the other. The essential conditions are that $\frac{W}{A}$ shall not be so decreased on the windward side as to be obliterated, and tension set up; and that it shall not be increased on the leeward side to an intensity greater than the material will stand. Taking the case of the buttress and wall shown in Fig. 75, subject to a bending moment from an inclined load. The centre of gravity of the weight of the section is found by ordinary calculation to be 0.92 ft. from the inner edge of the wall. The total vertical component of the weight of the brickwork and truss is 274 cwt., the area of the section is 15.8 sq. ft., and the intensity of vertical load $\frac{W}{A}$ is therefore $\frac{274}{15.8} = 17.35$ cwt. per sq. ft. The clockwise bending moment (M) of the lateral force is 274×0.66 ft. or 108.1 ft.-cwt., which tends to increase $\frac{W}{A}$ on the outside edge of the buttress and to decrease it on the inner side of the wall, the maximum pressure on the outside being $\frac{W}{A} + \frac{M}{Z}$, the minimum pressure $\frac{W}{A} - \frac{M}{Z}$. If the tensional intensity represented by $\frac{M}{Z}$ is greater than $\frac{W}{A}$ then tension is set up. The moment of inertia of the section is as follows :—

$$c\left\{\begin{array}{lll} \text{I of top of table} & = \frac{11 \times 0.92^3}{12} & = 0.712 \\ \text{I of bottom} & = \frac{11 \times 0.205^3}{12} & = 0.008 \\ \text{I of web} & = \frac{1.5 \times 2.225^3}{12} & = \underline{1.430} \end{array}\right.$$

$$ \qquad \qquad \qquad \qquad 2.150$$

$$d\left\{\begin{array}{ll} 11 \times 0.92 \times 0.46^2 & = 2.300 \\ 11 \times 0.205 \times 0.103^2 & = 0.040 \\ 2.25 \times 1.5 \times (1.125 + 0.205)^2 & = \underline{4.450} \end{array}\right.$$

$$\underline{\underline{I = 8.940}}$$

The two values for Z are therefore, as regards the inner edge, $\frac{8.94}{0.92} = 9.7$, and as regards the outer edge $\frac{8.94}{2.455} = 3.65$.

The extreme pressures are therefore :—

$$\text{Maximum } \frac{W}{A} + \frac{M}{Z} = 17.35 + \frac{108.1}{3.65} = 17.35 + 32.5$$

$= 49.85$ cwt. per square foot compression, and

$$\text{Minimum } \frac{W}{A} - \frac{M}{Z} = 17.35 - \frac{108.1}{9.3} = 17.35 - 11.3$$
$$= 6 \text{ cwt. per square foot.}$$

No tension is therefore set up, and the maximum pressure is not unreasonable for good brickwork.

DEDUCTION OF STRENGTH OF JOISTS FROM STRENGTH UNDER DISTRIBUTED LOAD.—The lists of many merchants of rolled joists still give only the distributed load which they will carry over different spans. This information can easily be utilised to ascertain the strength of the same joists under any loading if the fundamental fact of B.M. = M.R. be borne in mind.

For instance, suppose a given system of irregular loading will impose on a girder 10 ft. span a maximum B.M. of 100 in.-tons, and a joist list says that one joist (*a*) will safely carry a distributed load of 5 tons over 10 ft., whilst another (*b*) will take 7 tons.

The B.M. under an equally distributed load always equals $\frac{WL}{8}$, and the M.R. always equals the B.M. Therefore, the M.R. which the joist (*a*) can safely be called upon to exert is—

$$\frac{WL}{8} = \frac{5 \text{ tons} \times 120 \text{ in.}}{8} = 75 \text{ in.-tons,}$$

which is less than what is required.

Joist (*b*) can provide a M.R. of—

$$\frac{7 \times 120}{8} = 101 \text{ in.-tons,}$$

which is sufficient for the irregular loading under consideration.

CHAPTER XIII

SHEAR

Importance of Shearing Stresses—Simultaneous Action of Vertical Shear at Different Points—Examples of Stresses and Strains Appearing Simultaneously in Different Parts of a Structure but all Generated by One Pair of Forces—Examination of the Stresses in a Built-up Frame Cantilever.

IMPORTANCE OF SHEAR STRESSES.—Those portions of structural design which demand a clear comprehension of the action of shear stresses frequently prove to be those which most clearly bring home to the young designer the deficiencies of his previous studies. The reason for this may possibly be found in the fact that the majority of text-books succeed in conveying the impression that shear is one of those mysteriously intricate matters which are of very little importance in practical design, and which require, therefore, only a comparatively brief and cursory investigation.

This may be true with regard to wooden beams and rolled steel joists, which have so large an excess of material with which to resist the greatest shear stresses which can possibly come upon them that the calculation of those stresses in their case would be absurd. But in plate and lattice girders and in reinforced concrete beams the shear stresses are of the greatest possible importance, and affect the design and weight to a very marked degree. Many a design which has reached a fairly advanced stage has been discarded on account of the excessive shear stresses involved, and the author has more than once found in reconstructing old railway bridges for heavier axle loads that the whole question of whether a bridge could or could not be reconstructed without buying out expensive interests, turned entirely upon the possibility or otherwise of meeting those stresses. To regard them as being of little or no practical importance is a most dangerous view, and their apparent intricacy arises merely from the fact that they belong to the numerous category of forces which, like reactions, are seldom appreciated properly because evidences of their existence are seldom seen. For the same reason they are very liable to be overlooked altogether, and it is only common prudence to investigate their nature with particular care and in practice to invest them with, if anything, even more than their due importance.

Simultaneous Action of Vertical Shear at Different Points.—In the case of the girder shown in Fig. 76 (*a*) and in the cantilever at (*b*) the tendency of a load to bodily shear away the section which immediately carries it is obvious. It will be equally obvious when those sections are studied in their essential skeleton forms of braced

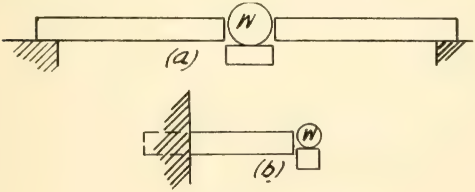


Fig. 76.

frames that the action of the web, whether it be an entire plate or single or double diagonal members, would be to attempt to turn the web in a diagonal direction or distort it into a diagonal shape and thus convert the vertical shear into horizontal flange stresses. But it is not so apparent that after the vertical shear has been thus converted into horizontal flange stress, it can immediately *reappear* in the *next* section as a fresh vertical shear *of the same amount* as before and proceed to act all over again on that section. This is seen in the divided cantilever of Fig. 77, and the state of balance shown there appears at first sight to be almost a paradox.

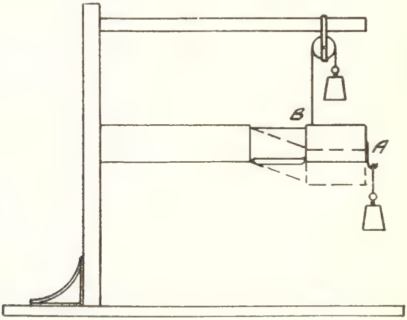


FIG. 77.

It would seem that when the web of the severed section had twisted round the vertical shearing stress due to the load into horizontal tension and compression in the flanges, that the capabilities of that stress would be exhausted, and all that would be required from the fixed portion of the cantilever would be a link at the top capable of resisting tension, and a strut at the bottom capable of resisting compression. But when such an arrangement is tried, it is found that without an upward force at B equal to the load, the severed structure tends to assume the form shown in dotted lines, and to collapse. The upward force may be provided by the pull of an inclined tie, or by the thrust of an inclined strut, or by a solid web instead of a counterweight, but it must be provided for somehow; for the section which is removed would be attacked by a shearing stress set up by the load in precisely the same way as the end section; and in addition it has to support the flange stresses generated in that end section by the load. In spite of the shear at A having been twisted round into the horizontal flange stresses, whose presence and amount is clearly indicated when tension and compression balances are introduced into the model, it *reappears* at B ready to act all over again with undiminished force upon the next section.

EXAMPLES OF STRESSES AND STRAINS APPEARING SIMULTANEOUSLY IN DIFFERENT PARTS OF A STRUCTURE, BUT ALL GENERATED BY ONE PAIR OF FORCES.—It is not always easy to grasp the possibility of the existence of several different stresses and strains acting simultaneously on different parts of the structure, and all caused by *one* pair of generating forces. A little consideration of the results obtained in the following experiments should, however, make it clear.

If five spring balances be hung parallel to each other with their hooks carefully adjusted at the same

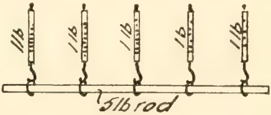


FIG. 78.

level, as in Fig. 78, then a rod weighing 5 lbs. strung on all the hooks, and bearing equally on all of them, will cause each balance to indicate 1 lb. But if the balances be hung one upon another in a chain, then the addition of a 5-lb. weight to the bottom balance will cause *each* balance to register a strain of 5 lbs. instead of 1 lb. as before; these strains being, of course, additional to those caused by the weight of the balances. If a weight of, say, $\frac{1}{2}$ lb., when hung on a thick rubber ring, has the effect of dragging it from a circle into an ellipse, then the effect of a weight of $\frac{1}{2}$ lb. hung on a chain of twenty such circular links will similarly elongate *every one of them* into the same elliptical form. As soon as the weight has acted upon and distorted the bottom ring, it is free to attack the next with undiminished energy, for it can act through the distorted ring as if the latter were a rigid body. The internal stresses in the bottom link can, in fact, only pull on the weight by virtue of their ability to simultaneously pull on the link above. The upper links are not relieved by the distortion of the lower ones, any more than a man at the bottom of a scrimmage is at all relieved of the weight of the top man of the heap by the fact of some intermediate bodies, between him and the top, being simultaneously in a state of compression. It will therefore not be surprising that a load on the end of a cantilever will create a downward shearing force, not merely at the end *but at every point along the whole length.*

The section which directly supports the weight hangs on the next section, and so on—each section tending to drag down its immediate neighbour.

If a force attacks a structure at one point, and is balanced by a contrary force or reaction applied to a different point, then *all* intermediate parts of the structure which *can* be stressed by the pair of forces *are* so stressed. In the bell-crank lever shown in Fig. 24, page 66, the vertical weight on the long arm is balanced by a horizontal pull on the short arm. But at the same time the pivot pin is pushed against the bearings in two directions, horizontally and vertically, and if the bearing be unable to resist those thrusts, or their resultant, it would fail. The same pair of balanced forces which thus stress the pin bearings are simultaneously tending to break the lever arms across, as well as trying to shear them through vertically, horizontally, and in an inclined direction; in fact to stress the whole structure simultaneously in every way possible to such forces.

EXAMINATION OF THE STRESSES IN FRAME CANTILEVERS.—If a cantilever be built up, as shown in Fig. 79, of hinged members into which spring balances are introduced which will measure the nature and amount of the stress in each member, it will first be found that either an inclined strut or an inclined tie is necessary to prevent the frame from

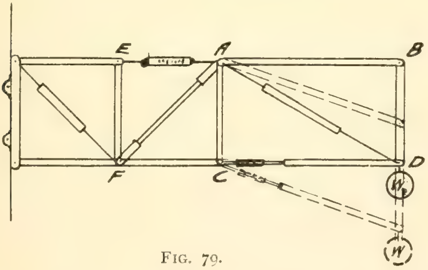


FIG. 79.

collapsing in the direction indicated by the dotted lines. Both a tie *and* a strut are not required, but one or the other is absolutely essential.

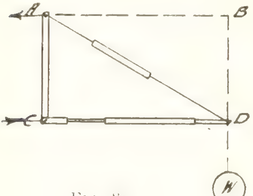


FIG. 80.

Considering the cantilever frame in Fig. 80 with an inclined tie it will be found that the upper horizontal member AB and the outer vertical member BD are under no stress—and that the remaining members are under the same stress as those which are found in a bell-crank lever of the same dimensions, viz., the bottom member exerts a horizontal push on the pivot C equal to the pull which would be required at A to balance the moment W × CD, and that the pull in AD is equal to the resultant of this push and the weight W. Now let the frame be joined on to several other similar frames, as in Fig. 81, so as to form one long cantilever.

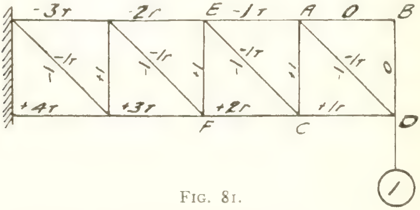


FIG. 81.

It will be recollected that the pivot of a bell-crank lever is subjected to horizontal and vertical stresses equal to the forces applied to the arms. The horizontal stress is the compression along CD which was found in the single frame, and the vertical stress is the reappearance of the vertical shear at A C. The extent of the horizontal force which would have been necessary at A to balance a bell-crank lever A C D would be $1 \times \frac{\mathrm{CD}}{\mathrm{AC}}$. If this ratio $\frac{\mathrm{CD}}{\mathrm{AC}}$ be termed r the horizontal stress due to weight 1 would be $1\,r$.

The fact that the weight 1 is free to act all over again at A C causes a fresh set of horizontal stresses equal to $1\,r$ to be generated in the second panel E A C F which it must carry in addition to those transmitted from the end panel. The third panel has to suffer a further increment, and so on.

It will be noticed that under a single concentrated load the vertical shear due to that load remains constant — but that the flange stress (which in parallel cantilevers is proportional to the bending moment) increases from nothing at B D to a maximum at the bearing. The difference between the top and bottom flange stresses is due to the type of bracing shown. Had the inclination of the diagonal bars been reversed, as Fig. 82, the difference would have been

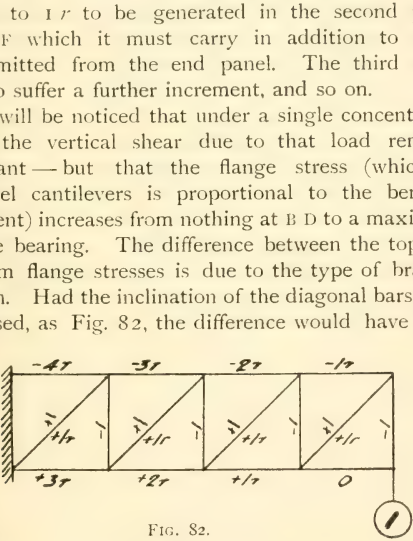


FIG. 82.

similarly reversed. With double diagonal bars or a plate or solid web, the flange stresses would have been equalised.

CHAPTER XIV

SHEAR *(continued)*

Vertical Shear in Cantilevers Supporting several Concentrated Loads and Distributed Loads—Relation between Vertical Shear in Girders and Cantilevers—Graphic Methods of Determining Vertical Shear—Generation of Flange Stress from Vertical Shear in Solid Web Girders and Cantilevers—Deduction of Vertical and Horizontal Shear from Bending Moment Diagram—Relation between Horizontal and Vertical Shear—Thickening of Web to Enable it to Act as a Diagonal Strut Unnecessary—Shear due to Rolling Loads—Examples of Shear and Rivet Calculations.

VERTICAL SHEAR IN CANTILEVERS SUPPORTING SEVERAL CONCENTRATED LOADS.—The vertical shear at any point in a cantilever A B, Fig. 83, due to a single concentrated load at B is equal to that load. Additional loads, as C and D, do not affect the shear to the right, but will each add an increment equal to itself to the shear on the left. The shear

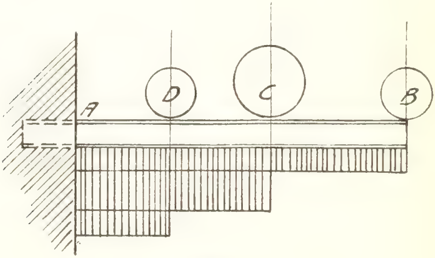


FIG. 83.

at any point in a cantilever can, therefore, be obtained by deducting from the total shear at the bearing any weights lying between that point and the bearing. If the total load were subdivided, as in Fig. 84, the

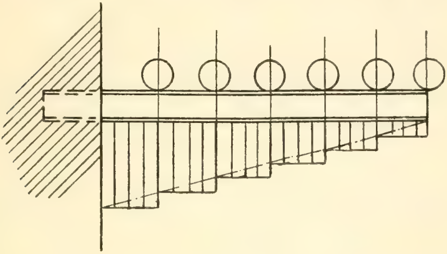


Fig. 84.

steps in the shear diagram would be less pronounced, until with an equally distributed load, such as the weight of the cantilever, they would subside into a straight line and the diagram of shear would be a triangle.

Relation between Vertical Shear in Girders and Cantilevers.—The vertical shearing

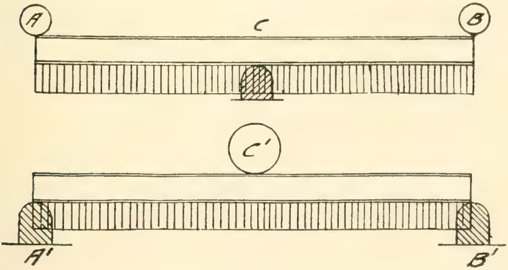


Fig. 85.

stress in a girder under concentrated load can be readily grasped by imagining the balanced double cantilever A B in Fig. 85 or 86 turned upside down.

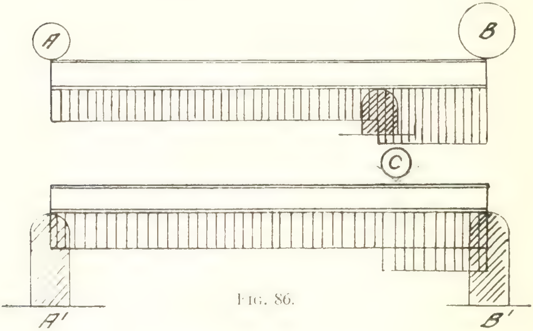


FIG. 86.

Neglecting the weight of the cantilever, the shear at any point in the cantilever A C will be equal to the load at A, and the shear at any point in B C will be equal to the load at B. The pressure on C will equal the two loads at A and B. Similarly in the girder A′ B′ loaded with a weight at C′ equal to the pressure on C, the shear at any point between A′ and C′ will be equal to the reaction at A, and the shear at any point between B′ and C′ will be equal to the reaction at B′.

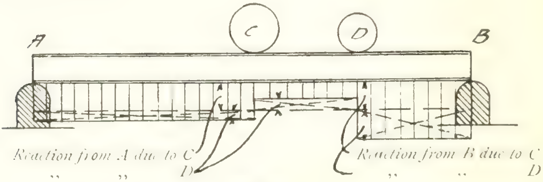


FIG. 87.

When a second load is imposed, as in Fig. 87, the reactions at A and B, and therefore the vertical shear from A to C and from B to D, will be increased, but between C and D the shear will be reduced. From A to C both weights call up reactions from A tending to cause clockwise rotation, and from D to B both weights similarly call up reactions from B tending to cause anti-clockwise rotation. But from C to D the weight C is calling up an anti-clockwise reaction from B, and the weight D is calling up a clockwise reaction from A, and the two tend to neutralise each other. In the divided model beam shown in Fig. 88, it will be found that when the weights and distances be so arranged that the reaction R B C equals the reaction R A D, there is no need for any inclined tie or strut in any section *e f* between C and D, but if the reaction R A D be greater than the reaction R B C then *e* would tend to rise and *f* to drop and a strut or tie would be required as shown. If the reaction R B C were the greater, then the strut or tie would require to be reversed or the whole would collapse.

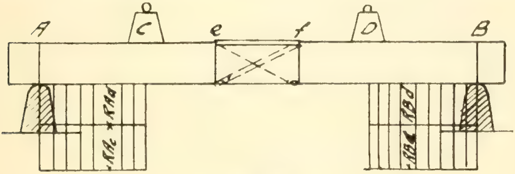


FIG. 88.

GRAPHIC METHODS OF DETERMINING VERTICAL SHEAR.—The calculation of the vertical shear at any point caused by a number of different vertical loads would, of course, be a very tedious operation, and it can be simplified, as far as concentrated loads are concerned, by the graphic method shown in Fig. 89. The end reactions having been found by funicular polygon or otherwise, the loads and reactions are set up vertically and horizontal lines are drawn through each point on the load line and vertical lines through each load and bearing. The lengths intercepted between the horizontal lines are the vertical shear stresses acting over each portion of the span. This method does not include shear due to the distributed load of the weight of the girder, which will tend to increase the end shears and to reduce the remainder. Its effect can be approximated by dividing the span up into equal short lengths and treating the weight of each panel as a concentrated load. But both the horizontal

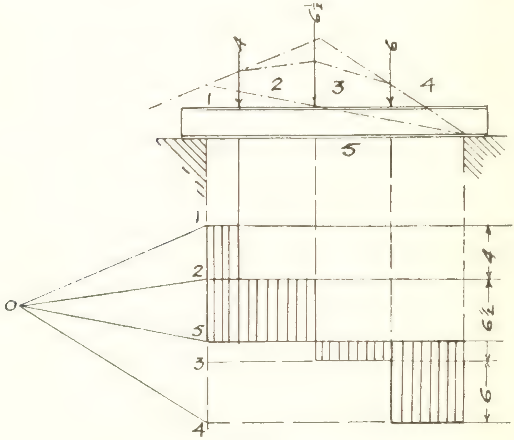


FIG. 89.

and vertical shears in parallel girders and cantilevers can be deduced exactly from the bending moment diagram in the following manner.

GENERATION OF FLANGE STRESS FROM VERTICAL SHEAR IN SOLID WEB CANTILEVERS AND GIRDERS.—In the thin webbed parallel cantilever A B, shown in Fig. 90, the bending moment increases from nothing at the end B to a maximum at the bearing A. The flange stress at any point is equal to the bending moment at that point divided by the effective depth. The effective depth being the same throughout, the diagram of bending moments represents also a diagram of flange stresses × the effective depth between the flanges. The flange stress at D is represented by the ordinate *d d*, and the flange stress at C by the ordinate *c c*. The flange A D is therefore being pulled to the left at C by a greater force than is pulling it to the right at D, and the difference between the two *e c* must be supplied by some other force pulling to the right between C and D. But

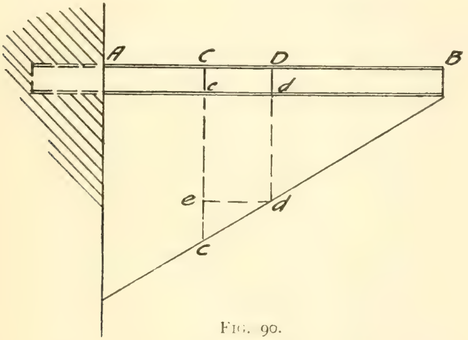


FIG. 90.

as nothing is touching the flange between C and D except the web, it is obviously some force transmitted by the web and equal to *e c* which is dragging on the flange between C and D. *All flange stresses are in fact thus generated in the web, whether in girders or cantilevers*, a girder being only an inverted double form of cantilever. This becomes abundantly clear if the portion between C and D be considered independently, as in Fig. 91. The weight W may not be directly resting on this panel, but the portion D B of the cantilever, together with the weight, has to be carried, and the panel C D has to carry it as

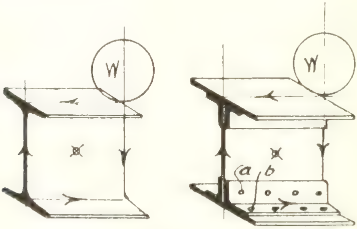


FIG. 91.

if it were a load, as shown. The tendency of this load is to rotate the panel about the central point of the neutral axis in a clockwise direction. But it cannot turn in a clockwise direction because the flanges hold it back. The flanges keep it back because, and only because, they are firmly attached to the web, and if the web were able to shear itself away from the flanges it would do so. Riveted plate girders and cantilevers are quite liable to fail in this way through the web being torn by the rivets which connect it to the flange angles, and the first consideration in the design of a plate girder is the thickness of web and the pitch of riveting which will resist this tendency.

In skeleton framed girders the relation between vertical and horizontal shear is determined by the spacing of the vertical posts and the inclination of the diagonal ties or struts. In thin webbed plate girders of ordinary construction the stiffeners as a rule are amply substantial as posts to take the vertical shear, and the most important consideration is the varying horizontal shear between the web and the flanges.

DEDUCTION OF VERTICAL AND HORIZONTAL SHEAR FROM BENDING MOMENT DIAGRAM.—In Fig. 90 the ordinate $ec \div$ the depth of the section represents the additional flange stress supplied to the flanges by the web between C and D. The *average* intensity per foot run of horizontal drag between the two over that length CD would be represented by $\frac{ec}{\text{CD} \times \text{effective depth}}$. The diameter of rivets and the number per foot run (*i.e.*, the pitch) necessary to resist this drag can readily be calculated. This method of ascertaining from the bending moment diagram the amount, and also the average intensity, of horizontal shear, is equally applicable to girders. The amount of the vertical shear can be obtained from it by multiplying by the ratio of the depth of the panel selected for consideration to its length.

RELATION BETWEEN VERTICAL AND HORIZONTAL SHEAR.—Lattice girders are divided into definite panels by the members. In plate girders of ordinary construction the vertical stiffeners act as posts and divide the plate web into panels which act as inclined struts or ties, the length of each strut or tie being equal to the diagonal of each panel, but their width is of course unnecessarily large. When the panels are square, the vertical shear will be equal to the horizontal shear, as it would be in the case of a skeleton or framed girder with square panels and ties or struts inclined at an angle of 45°. If the ties or struts be inclined at a steeper or flatter angle than 45°, then the increment of flange stress supplied by each inclined strut or tie is greater or less than the vertical shear according to the proportion of the length to the depth of each panel.

THICKENING OF THE WEB TO ENABLE IT TO ACT AS A DIAGONAL STRUT UNNECESSARY.—In this connection it may be noted that a plate web can act as a large tie in one diagonal direction, or as a large strut in the opposite diagonal direction, or both; but there is no need for both a strut and a tie in any one panel, and a plate web can only fail by crumpling in its capacity as a strut after it has failed as a tie, as will be seen in the skeleton frame in Fig. 92.

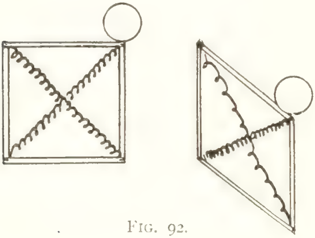


FIG. 92.

It is frequently suggested that every plate web should be designed so as to be capable of acting as a diagonal strut, but this is obviously unnecessary as long as it can act as a tie, and the thickness necessary for stiffness as a diagonal strut also adds largely to the weight without any compensating advantage, and it is not done in the best practice. The thickness necessary to prevent the flange rivets from tearing into the web enables a plate to act to some extent as a diagonal strut, and this tends to relieve the calculated pressure on the rivets. It is not customary to make any allowance for this in practical design, and in addition, the maximum horizontal shear in any panel is considered as existing throughout that panel; but on the other hand the horizontal shear is frequently deduced from the vertical and taken as being equal to it at all points, although the stiffeners near the middle of the span are often spaced at a distance apart greater than the depth of the girder.

Rectangular wood beams and rolled joists have necessarily thick webs which do not require additional struts, and which, therefore, consist of panels of indefinite length. The determination of the relation between vertical and horizontal shear in such webs (which is still the subject of experimental research) is of little importance in practical design, because the intensity of horizontal shear, which determines the riveting of extra flange plates, can always be ascertained from the bending moment diagram.

SHEAR DUE TO ROLLING LOADS.—If the vertical shear on any point of a girder A B, Fig. 93, due to a

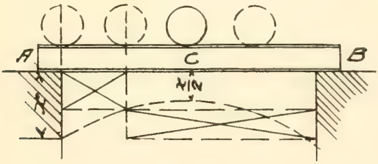


FIG. 93.

single rolling load X, be calculated for a number of different positions of that load and plotted in the form of ordinates, they would be found to be in a parabolic curve, the ordinates at A and C being X and $\frac{X}{2}$ respectively.

In the same girder, Fig. 94, under a train of rolling loads amounting to a total weight of W when the span is fully covered, the maximum shear at the bearing would be $\frac{W}{2}$, and the position of the load which imposes the maximum shear at any point will be found to be when the head of the train is over that point; at the centre this will represent a shear of $\frac{W}{8}$, and the diagram of maximum shears will be a parabolic curve intercepting ordinates of $\frac{W}{2}$ at the bearings and $\frac{W}{8}$ at the centre.

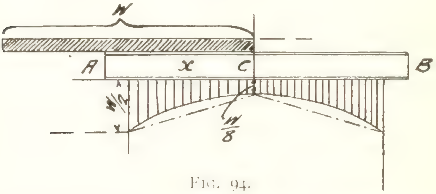


FIG. 94.

The calculation of the ordinates of parabolic curves is a tedious operation, and the shear due to a distributed rolling load can be determined far more easily and with far less chance of error by assuming the line of maximum shears to be the straight dot-and-dash line shown in Fig. 94, *i.e.*, that a shear of $\frac{W}{8}$ occurs at all points of the span with an additional shear, which varies from $\left(\frac{W}{2} - \frac{W}{8}\right) = \frac{3W}{8}$ at the bearing to O in the centre. The proportion of $\frac{3W}{8}$ acting at any point, x, would then vary with the distance xc of that point from the centre, and the total shear at x would be represented by $\frac{W}{8} + \left(\frac{3}{8}W \times \frac{XC}{AC}\right)$.

The diagram of a distributed load being a similar triangle, the load on a girder due to its own weight can therefore be added to a dead load equivalent to the rolling load, and both be calculated together. The error involved is on the safe side, but it is so slight as to be of no practical importance.

EXAMPLES OF SHEAR AND RIVET CALCULATIONS.—The foregoing principles are used in solving many problems of practical design, but they are most frequently employed in determining the stress in the diagonal members of lattice girders, the thickness of webs in plate girders, and the riveting in plate and compound girders.

It will be clear from the consideration of the stresses in the skeleton cantilever in Fig. 79, page 161, that the stress in any tie or strut of a lattice girder or cantilever would be represented by the diagonal of a parallelogram, of which the adjacent

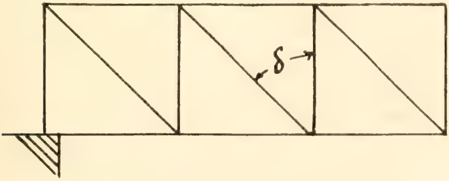


FIG. 95.

sides represented the vertical and horizontal shear stress occurring at the point where the flange meets that end of the inclined member which is nearest the bearing of a girder, or the free end of a cantilever. Trigonometrically, it would be represented by the vertical shear, Fig. 95 × sec. δ, δ being the angle which the bar makes with the vertical.

The riveting of a typical plate girder, as Fig. 97, consists of—

(1) The main riveting between the (*a*) flange angles and the web, and (*b*) between the various plates and angles which form the flanges.

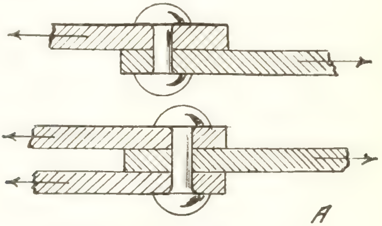


Fig. 96.

(2) The secondary riveting in the joints, if any, in the web of flange plates, and the rivets which connect the stiffeners and gussets, if any, to the web and flanges.

Rivets of a diameter d subjected to single shear, as Fig. 96, present a cross-sectional area of $d^2 \times .7854$ to failure. Against double shear two such surfaces would be opposed. Against bearing (or the tendency of a rivet or a plate to bite into each other) there is only available a surface equal to the thickness of the plate × the *diameter* of the rivet, *not* the circumference, a 1-in. cylindrical rivet presenting the same effective surface for this purpose as a 1-in. rivet of square section.

The great majority of rivets used in building up plate girders are $\frac{3}{4}$ in. diameter, $\frac{7}{8}$-in. rivets are used but seldom, and 1-in. only to resist very exceptional shear stresses. Owing to the use of multiple punching and drilling machines, the pitches in ordinary use are confined, as far as possible, to 3 in. or 4 in. or multiples thereof, as 6 in., 8 in., and 2 in. zigzag.

Considering the riveting in the main flange angles of a plate girder, Fig. 97, it is obvious that at any section the two vertical rivets *b* through the flange plates in single shear will be, as far as failure by shearing is concerned, equal in strength to the single horizontal rivet *a* through the web, which is in double shear. It is also obvious that in the case of all web plates less than the abnormal thickness of 1 in. the horizontal rivets will be more liable to fail by bearing than by double shear. The thickness of web plates is, therefore, determined by ascertaining the intensity of horizontal shear per foot run, and then selecting a thickness of plate which will, with $\frac{3}{4}$-in. rivets at a practical pitch, keep down the stress in bearing per square inch within safe limits.

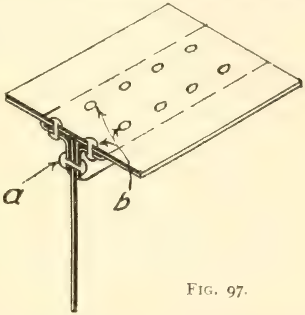


Fig. 97.

The following table gives the area of material called into play per foot run to resist bearing with ordinary plate thicknesses and rivet pitches.

Diameter of Rivets.	Pitch of Rivets	Thickness of Web.	Bearing Area per Foot Run.
in.	in.	in.	sq. in.
$\frac{3}{4}$	4	$\frac{3}{8}$	0.84
,,	,,	$\frac{7}{16}$	0.98
,,	,,	$\frac{1}{2}$	1.12
,,	,,	$\frac{9}{16}$	1.26
,,	,,	$\frac{5}{8}$	1.40
$\frac{3}{4}$	3	$\frac{3}{8}$	1.12
,,	,,	$\frac{7}{16}$	1.31
,,	,,	$\frac{1}{2}$	1.50
,,	,,	$\frac{9}{16}$	1.68
,,	,,	$\frac{5}{8}$	1.87
$\frac{7}{8}$	4	$\frac{3}{8}$	0.98
,,	,,	$\frac{7}{16}$	1.15
,,	,,	$\frac{1}{2}$	1.31
,,	,,	$\frac{9}{16}$	1.48
,,	,,	$\frac{5}{8}$	1.64
$\frac{7}{8}$	3	$\frac{3}{8}$	1.31
,,	,,	$\frac{7}{16}$	1.53
,,	,,	$\frac{1}{2}$	1.75
,,	,,	$\frac{9}{16}$	1.97
,,	,,	$\frac{5}{8}$	2.18
$\frac{7}{8}$	2 in. zigzag	$\frac{5}{8}$	3.28
,,	,, ,,	$\frac{11}{16}$	3.61
,,	,, ,,	$\frac{3}{4}$	3.94

The intensity of stress per foot run can either be ascertained from the bending moment diagram as before described, or it can be deduced by dividing the vertical shear at the worst end of any bay by the effective depth in feet, and assuming the horizontal intensity to be equal to it. For example, a girder 4 ft. deep, end reactions 40 tons, safe allowable stress per square inch in bearing 7.5 tons, intensity of shear at end panel $\frac{40}{4} = 10$ tons per foot run. Area required against failure by bearing $\frac{10}{7.5} = 1.3$ sq. in., which would be supplied by a $\frac{7}{16}$-in. web, and $\frac{3}{4}$ in. rivets, 3 in. pitch, or by a $\frac{5}{8}$-in. web and $\frac{3}{4}$-in. rivets, 4 in. pitch, whichever were the more convenient.

The rivets between the various plates of a built-up flange are usually safe from failure by bearing, and the customary 4 in. pitch almost invariably renders them safe against failure by shear. The rivets in joint plates of flanges are assumed to be in single shear, and the joint plates are made long enough to pick up sufficient rivets in single shear to transmit the strength of each plate (in tension or compression as the case may be) over to the plate to which it is joined. Rivets in stiffeners, etc., are usually spaced at the standard pitch of 4 in. In riveting additional flange plates on to rolled joists to form so-called "compound" girders, it must be remembered that each element of the flange takes its own share of the flange stress, and, therefore, the single shear on the rivets on the outside plates is caused by only a proportion of the total shear intensity as obtained from the bending moment diagram. It will be found that 10 in. or even 12 in. pitch is sufficient in ordinary cases, but with a pitch of over 6 in. damp is likely to get between the two plates where they cannot be painted, and the resulting rusting causes swelling and tends to bulge the plates apart. This not only sets up secondary stresses which may burst off the rivet heads and cause compression flanges to buckle, but it also widens the aperture through which damp can enter and tends to aggravate the decay.

CHAPTER XV

DEFLECTION

Importance of Deflection Calculations—Stiffness no Criterion of Strength—Modulus of Elasticity—Constituent Factors of the Curvature of a Girder under Load—Relative Effect of Depth on Strength and Stiffness—Relative Effect of Length on Strength and Stiffness—Computation of Total Deflection of Cantilever—Cantilever Concentrated Load: Uniform Flange Stress—Cantilever Distributed Load: Uniform Flange Stress—Cantilever Concentrated Load: Uniform Section—Cantilever Distributed Load: Uniform Section.

IMPORTANCE OF DEFLECTION CALCULATIONS.—The practical designer wishing to understand the important subject of deflection is met in nearly all text-books with the statement that the subject is beyond his intelligence, and that he must trust blindly to the formulæ and rules supplied to him. As these formulæ usually involve the calculation of Moments of Inertia he seldom uses them, and therefore probably fails to note the small arithmetical differences in them, which indicate that the *stiffness* of a beam or cantilever bears little or no relation to its *strength*. Failure to appreciate the importance of the essential factors regulating stiffness frequently leads, on the one hand, to undue confidence in the strength of a beam or member which proves to be stiff under its load, or on the other hand it may lead to unexpected and disfiguring deflections occurring in members whose strength may have been quite correctly predetermined.

If the factors governing deflection be investigated with a little care and a very little common-sense, the subject will be found to present no more difficulty than the mechanics of a child's see-saw.

STIFFNESS NO CRITERION OF STRENGTH.—It is well known that the strength of a girder or cantilever varies directly according to its fibre strength (f), multiplied by its muscular development or Section Modulus Z (which is roughly its effective flange area a multiplied by its effective depth d, or $a\ d$); whilst its length (l) directly decreases its strength, by increasing any bending moment or

$$\text{Strength} \propto \frac{ad}{l} \times f,$$

and in the case of beams of rectangular section whose effective flange area a is always $\frac{bd}{4}$, and whose effective depth is always $\frac{2}{3}d$, then

$$\text{Strength} \propto \frac{bd}{4} \times \frac{2d}{3} \times \frac{f}{l} = \frac{bd^2}{6} \times \frac{f}{l}.$$

The deflection of a girder or cantilever being merely the result of the "give" of the flange fibres under stress, one would expect the stiffness to vary, like the strength, with the flange area, effective depth, and length. It certainly does vary with the flange area, *i.e.*, with the flange stress, but the depth and length affect it very differently.

If the flange stress in a girder or cantilever be reduced by halving the load, the deflection will be accordingly reduced by one half, as would be expected. But if the flange stress be reduced to one half by halving the span, the deflection ($\triangle$) will be reduced to only $\frac{1}{8}$ of its previous amount instead of $\frac{1}{2}$, and generally the deflection varies, not with the span, but with the cube of the span, or, $\triangle \propto l^3$.

Similarly, if the flange stress be reduced by doubling the flange area, the deflection is halved; but if the flange stress be reduced by doubling the depth, then the deflection will be reduced, not to $\frac{1}{2}$ but to $\frac{1}{4}$, the deflection varying inversely with the square of the depth, or, $\triangle \propto \frac{l^3}{d^2}$, and in girders of rectangular section, where doubling the depth also doubles the effective flange area $\left(\frac{bd}{4}\right)$, then the deflection would be reduced to $\frac{1}{8}$, viz., as $\frac{l^3}{d^3}$.

Still more strange is the want of connection between the stiffness of a beam and the fibre strength (f) of its material. One would expect, for instance, a beam of oak or teak to be much stiffer than a beam of deal, whereas the exact reverse is the case—a beam of oak or teak, which will carry a distinctly heavier load than a beam of deal of the same size and span, will always deflect more under the same load. A cast-iron beam is far weaker than a steel or wrought-iron beam of the same size, but all three would be of practically the same stiffness when stressed within the elastic limit. Concrete also, although brittle, is within its small limit of elasticity far *more* flexible than steel. Obviously, therefore, the rigidity of a material is no criterion of its fibre strength, and must be separately studied.

MODULUS OF ELASTICITY.—The rigidity of a material is defined by what is known as its Modulus of Elasticity (E), an imaginary stress per square inch which would stretch the material to double its length or compress it to annihilation; as, for instance, an imaginary stress of 13,000 tons per square inch for mild steel. If, instead of this imaginary stress, steel be loaded with only 1 ton per square inch, then the stretch would be $\frac{1}{13{,}000}$ of the length of the bar operated on. The E. of a material, therefore, when expressed in tons per square inch, represents not only an imaginary stress, it also represents that proportion of the length of a stressed bar which it will stretch or compress under a unit stress of 1 ton per square inch. All deformations *within the elastic limit* can be easily deduced from this, and what takes place outside the elastic limit is of no practical consequence.

If the length of any bar be divided by the Modulus of Elasticity of its material in tons per square inch, the stretch under a stress of 1 ton per square inch is obtained ; and the stretch under any greater stress, such as 5 or 10 tons per square inch, is the same result multiplied by 5 or 10.

If a steel bar 13 ft. long were subjected to a direct tension or compression of 1 ton per square inch, it would elongate or shorten, as the case may be, to the extent of $\frac{13 \text{ ft.}}{13{,}000}$, or $\frac{1}{1000}$ of a foot. If it were loaded to the extent of 10 tons per square inch, the deformation would be 10 times that amount or $\frac{1}{100}$ of a foot. On the other hand, if the bar were 26 ft. long, the extension or compression under a load of 1 ton per square inch would be $\frac{26 \text{ ft.}}{13{,}000}$, or $\frac{1}{500}$ of a foot ; and if the load imposed a stress of 10 tons per square inch, the compression or stretching would be $\frac{1}{50}$ of a foot. The greater the stress the greater the deformation under that stress, and also the longer the bar, the greater will be the total deformation. If a piece of elastic 6 in. long stretches to the extent of 1 in. under a load of 1 lb., then a piece 12 in. long would stretch to the extent of 2 in., and a yard of the same elastic under the same load would stretch to 3 ft. 6 in.

The term Modulus of Elasticity is an unfortunate one. It is not a measure of the elasticity of a material, which is its power of recovering its original dimensions after the removal of any stress, nor is it a measure of its ductility—it is a measure of its *stiffness*.

If the moduli of different materials, as given in any good text-book, be compared, it will be found that they vary in quite a different manner to their respective *strengths*. Deal has a higher modulus than oak, teak, greenheart, or any hard wood. Concrete is much less than steel, whereas hard and mild steels and cast iron, which are widely different in strength, differ but little in stiffness. It is for this reason that there is but little to choose between the strengths of similar long columns of the three materials, which would all fail by bending; steel being commonly selected for such a purpose merely because it is the more reliable material.

CONSTITUENT FACTORS OF THE CURVATURE OF A GIRDER UNDER LOAD.—The curvature of a girder under load is caused almost entirely by the stretching of one flange and the compression of the other under horizontal flange stress, the deformation of the web affecting the general curvature to such a minute degree that it may be (and in practice is) disregarded.

The effect of such stretching and compression of the flanges can be most clearly seen in the case of a cantilever loaded with a single concentrated load, as in Fig. 98. Before the weight is applied, let it

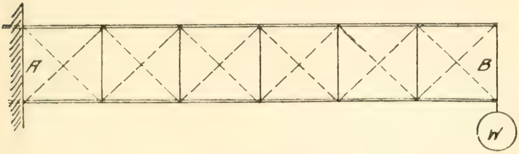


FIG. 98.

be assumed that the cantilever is divided up into a number of regular panels. Upon applying the weight, the upper part of each panel will be stretched,

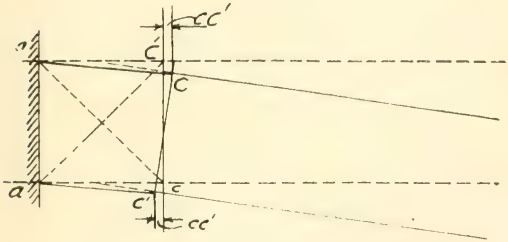


FIG. 99.

and the lower part will be compressed, so that each panel wlll assume a taper form resembling the voussoirs of an arch, as shown in Fig. 99.

The alteration in the length of a flange would be its length ÷ the modulus of elasticity × the stress per square inch, *i.e.*, its stretch under a stress of 1 ton per square inch multiplied by the stress it is actually receiving.

Let a diagram of flange stress per square inch be made, as in Fig. 100, in which A G represents the flange stress per square inch in each flange of the first panel. Then the upper bar of that panel will be extended and the lower bar compressed by an

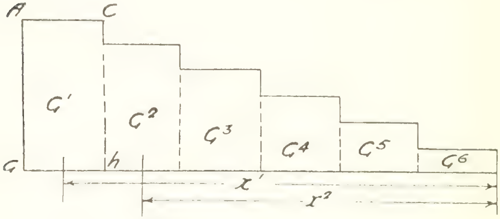


FIG. 100.

amount equal to the length (A C) of the panel ÷ the modulus of elasticity (E) × stress intensity per square inch (A C); or the alteration in length (C C′ in Fig. 99) $= \dfrac{AC \times AG}{E}$.

Of this, A G × A C will be represented by the *area* of the figure A G *h* C. If this area be called G′, then $\dfrac{G'}{E}$ will express either the elongation (C C′) of the upper bar, Fig. 99, or the compression ($c\,c'$) of the lower bar.

The altered form of the panel is shown in Fig. 99, and the *inclination* of the line C c', which was originally vertical, will be measured by the linear extension of the upper flange ÷ half the depth of the girder or by—

$$\frac{\text{Extension of AC} + \text{compression of } ac}{\text{depth of girder}}, \text{ or } \frac{2G}{Ed}$$

Even if the rest of the girder were subject to no further stress, this tapering of the first voussoir would have the effect of setting up a gradient of deflection which passes along the *whole length* of the cantilever. It will be seen that the longer the cantilever the greater will be the droop or deflection between the first panel and the end, due to that initial gradient or slope set up in the first panel ; also that the depth of the girder is one of the two essential factors which determine this initial gradient. If the extension C C and the compression *c c* were exercised in a deeper girder or cantilever, the initial gradient of the first panel or voussoir would be obviously less severe than if they were exercised in a shallow girder. This fact is very important, and merits some attention. Fig. 101 shows two cantilevers of the same span, and carrying loads which result in the same flange stress, and therefore the same stretch of the flange fibres C C. Yet if the deflection of the deeper cantilever were considered reasonable, then that of a shallow cantilever would appear to

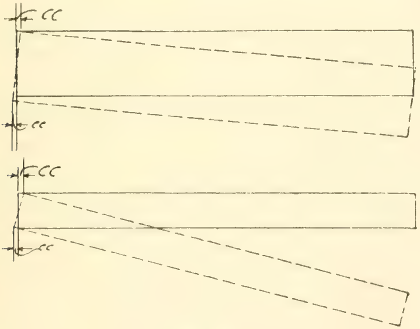


FIG. 101.

be excessive to a startling degree. A small deflection does not therefore necessarily indicate moderate flange stresses and safety, any more than a large deflection necessarily indicates danger. In a short deep girder or cantilever which deflects $\frac{1}{2}$ in. under its load, the fibres are possibly being punished to a far greater degree than those in a longer and shallower one which deflects 2 in. Deflection *per se* is no criterion of safety or danger unless it be considered in its proper relation to length and depth. Cases have been known where bridges which showed quite a small and apparently satisfactory deflection under a test live load of locomotives were really so distressed by that load that they soon afterwards quietly collapsed under the load of their own weight.

Although the deflection of a cantilever is the result of the stretching of one flange and the compressing of the other, its amount is also affected by the depth, which determines the initial gradient, and by the length, which determines the effect of that gradient, and also the amount of stretch.

RELATIVE EFFECT OF DEPTH UPON STRENGTH AND STIFFNESS.—It will now be clear why the depth of a girder has a greater effect in reducing deflection than it has in reducing flange stress. In reducing flange stress in a rectangular girder it acts twice over, first by increasing the area of the flange, and secondly by increasing the leverage at which that area acts, viz., $d \times d$ or d^2. In reducing the deflection, it acts first in that double fashion in determining the flange stress, and then exerts its influence again in determining the initial gradient, and so acts as $d^2 \times d$ or d^3 instead of d. Or in the case of thin-webbed girders, where it exerts no influence in the *area* of the flange, it acts as $d \times d$ or d^2 instead of d.

Relative Effect of Length on Strength and Stiffness.—The length also of a bended cantilever or girder, which has previously used its influence as *leverage* in setting up flange stress, acts over again with regard to deflection by multiplying the initial gradient, and operating, therefore, to increase the deflection, as $l \times l$ or l^2. It has, however, a further influence, because if we consider the remaining panels we find that each panel has its own particular flange stress, which in its turn has to be multiplied by the length of that panel, and sets up a further gradient, which contributes a further quota to the total deflection. So that the length acts over again in the same way that a 6-in. stretch takes place in a yard of loaded elastic as compared to 2 in. in a foot similarly loaded. The length, therefore, operates to increase deflection, first as leverage by increasing the flange stress, then to enhance at the end the camber of every voussoir, and then also by collecting the different stretches of every voussoir or panel under stress. So that whilst it only acts as l to enhance the flange stress, it acts as $l \times l \times l$ or l^3 to enhance the deflection.

Computation of Total Deflection of Cantilever.—The total deflection D D^6 in Fig. 102

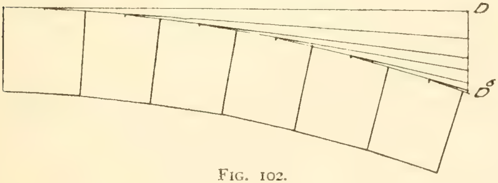


Fig. 102.

is, of course, the sum of all the deflections, each one of which is equal to the area of the stress diagram of each panel ÷ E *d* and multiplied by its distance from D, and the total deflection D D^6 *is equal to the area of the whole diagram × the distance of its centre gravity ÷ E d.* This gives the key to the whole problem of deflection of any *parallel* girder or cantilever. All we have to do is to imagine the diagram of flange stress in either flange per square inch as a load, and its moment about the point of deflection (in this case D) will represent the deflection due to it. As there are two flanges, and both are deforming under the stress, this must, of course, be multiplied by 2.

If, therefore, G represents the entire area of the diagram, and X the distance of its centre of gravity from B, the deflection will be $\frac{2GX}{ED}$.

This simple solution can now be applied to different classes of girders and cantilevers, with precisely the same results as are obtained by the more obscure mathematical processes.

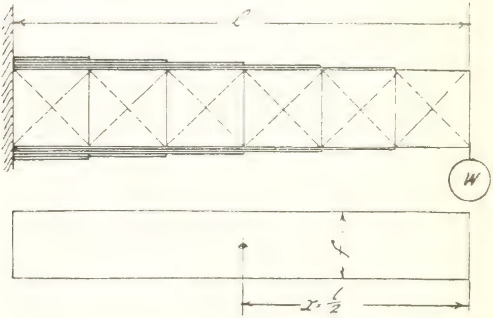


FIG. 103.

CANTILEVER CONCENTRATED LOAD: UNIFORM FLANGE STRESS.—First take a cantilever, in Fig. 103, with flanges proportioned to give a uniform flange stress. The flange stress diagram will be a rectangle—its area (G) will be fl, its leverage (X) will be $\frac{1}{2}l$, and $\frac{2GX}{Ed}$ will in this case be represented by $\frac{2fl \times \frac{1}{2}l}{Ed}$ or $\frac{fl^2}{Ed}$. If we want to represent this in terms of load (W), then $f =$ the bending moment ÷ the flange area and depth, or $\frac{Wl}{Ad}$, and the deflection $= \frac{Wl^3}{EAd^2}$.

CANTILEVER DISTRIBUTED LOAD: UNIFORM FLANGE STRESS.—If the load W had been distributed, then both f and the deflection would be half these values when expressed in terms of W or $\frac{Wl^3}{2EAd^2}$.

CANTILEVER CONCENTRATED LOAD: UNIFORM SECTION.—Next take a cantilever of uniform section,

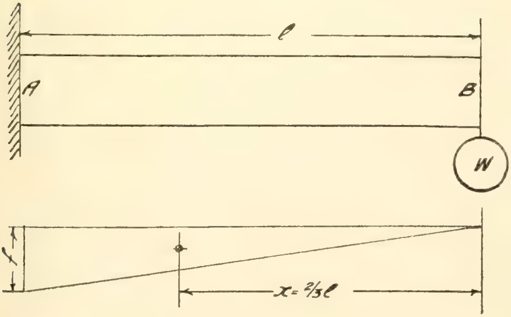


FIG. 104.

as Fig. 104, loaded at one end. The area of the stress diagram in this case will be $\frac{fl}{2}$, the distance of its centre of gravity from B will be $\frac{2l}{3}$, and $\frac{2GX}{Ed}$ will be represented by $\frac{2fl^2}{3Ed}$, or expressed in terms of the load W, $f = \frac{Wl}{Ad} = \frac{2Wl^3}{3EAd^2}$.

CANTILEVER DISTRIBUTED LOAD: UNIFORM SECTION.—Next take such a cantilever, loaded with a distributed load, as Fig. 105. In this case the diagram of stress intensity is traced by a parabola.

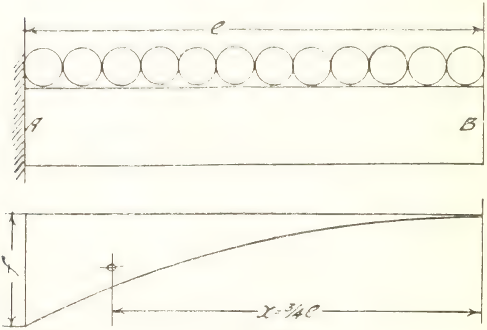
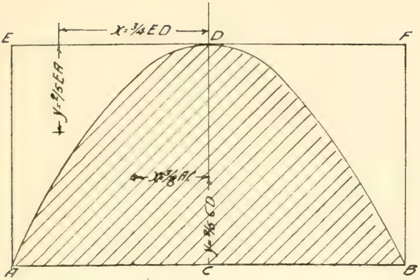


FIG. 105.

The area of it (obtained from the particulars given in Fig. 106) would be $\frac{2fl}{3}$, the distance of its centre of gravity would be $\frac{3l}{4}$, and $\frac{2GX}{Ed}$ would be $\frac{2fl}{3} \times \frac{3l}{4 \times Ed}$ or $\frac{6fl^2}{12Ed}$ or $\frac{fl^2}{2Ed}$. f in this case would be



Hatched Area, $\frac{2}{3}$ *EFBA*. *Unhatched Area*, $\frac{1}{3}$ *EFBA*.

FIG. 106.

$\frac{\text{WL}}{2\text{A}d}$, and the $\triangle$ for any distributed load W would be $\frac{\text{W}l^3}{4\text{EA}d^2}$.

Obviously the deflection in any kind of parallel cantilever can be expressed in simple terms of load, span, depth, flange area, and the modulus of elasticity of the particular material of which the cantilever is constructed.

CHAPTER XVI

DEFLECTION *(continued)*

Girder Uniform Section : Concentrated Load—Girder Uniform Section : Distributed Load—Girder Uniform Flange Stress : Distributed Load—Girder Uniform Flange Stress : Concentrated Load—Construction of Deflection Formulæ—Handy Formulæ for Deflection of Wooden Beams—Limiting Ratios of Depth to Span for given Ratios of Depth to Span—Effect of End Fixing on Deflection of Cantilevers.

GIRDER UNIFORM SECTION : CONCENTRATED LOAD.—Fig. 107 shows a girder under a concentrated load.

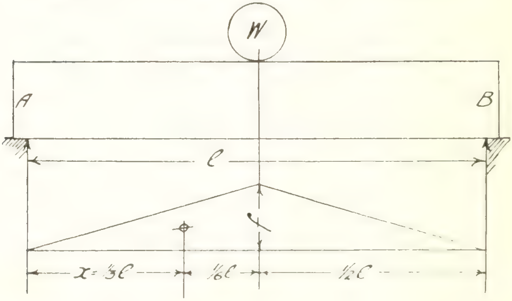


FIG. 107.

This may be considered as two cantilevers back to back, fixed in the centre against the downward bearing load, and loaded with the end reactions of $\frac{W}{2}$, the central deflection being equal to the deflections of either cantilever.

In this case the area of the stress diagram G $= f \times \frac{l}{2} \times \frac{1}{2}$, or $\frac{fl}{4}$. X, the distance of its centre of gravity from the point of greatest deflection *at the bearings*, would be $\frac{2}{3}$ of $\frac{1l}{2}$ or $\frac{1l}{3}$, and

$$\frac{2GX}{Ed} = \frac{2 \times \frac{fl}{4} \times \frac{l}{3}}{Ed} = \frac{fl^2}{6Ed}.$$

If $f = \frac{Wl}{4Ad}$, then the deflection expressed as before in terms of W $= \frac{Wl^3}{24EAd^2}$.

GIRDER UNIFORM SECTION : DISTRIBUTED LOAD.—The next case is that of a girder loaded with a distributed load (Fig. 108). Treating it in the same way as two cantilevers under a distributed load, we attribute the values in $\frac{2GX}{Ed}$ as before. G (the area of either half of the stress diagram) $= \frac{2}{3} \frac{fl}{2}$, and

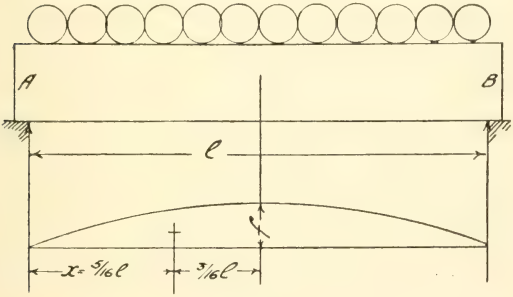


FIG. 108.

the distance of its centre of gravity from A (X) = $\frac{5l}{16}$

$$\frac{2GX}{Ed} = 2 \times \frac{2}{3}\frac{fl}{2} \times \frac{5l}{16Ed} = \frac{5fl^2}{24Ed^2}.$$

In this case $f = \frac{WL}{8Ad}$, so that the deflection in terms of W is

$$\triangle = \frac{5 \times Wl \times l^2}{24 \times 8 \times Ad \times Ed} = \frac{5Wl^3}{192EAd^2}.$$

GIRDER UNIFORM FLANGE STRESS: DISTRIBUTED LOAD.—The next case, Fig. 109, is that of a girder with flanges proportioned to the load, giving a uniform flange stress under a distributed load.

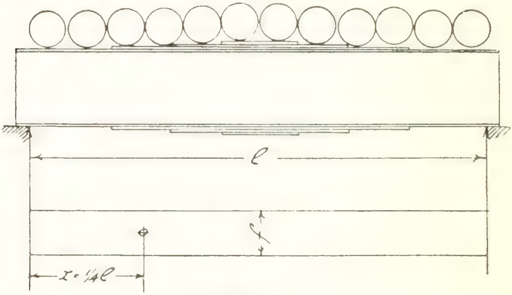


FIG. 109.

The values of $\frac{2GX}{Ed}$ will be as follows :—

$$G = \frac{fl}{2}. \quad X = \tfrac{1}{4}l. \quad f = \frac{Wl}{8Ad}.$$

$$\frac{2GX}{Ed} = \frac{fl \times l}{4Ed} = \frac{fl^2}{4Ed} = \frac{Wl^3}{32EAd^2}.$$

GIRDER UNIFORM FLANGE STRESS: CONCENTRATED LOAD.—It is obvious by inspection that in such a girder under a concentrated load the value of $\frac{2GX}{Ed}$ would be the same, viz. $\frac{fl^2}{4Ed}$, but as $f = \frac{Wl}{4Ad}$, then $\triangle = \frac{Wl^3}{16EAd^2}$.

CONSTRUCTION OF DEFLECTION FORMULÆ.—Each of the seven cases dealt with results in the term $\frac{Wl^3}{EAd^2}$, multiplied by the varying figures 1, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{1}{24}$, $\frac{5}{192}$, $\frac{1}{32}$, and $\frac{1}{16}$, which varying figures may be expressed as a constant C.

In the foregoing deductions the approximation A d (flange area × effective depth) has been used to express the section modulus for the sake of clearness. This should now be replaced by the more exact expression Z, so that the general formula would read $= \frac{Wl^3}{EZd} \times C$. The term d in this formula would be the full depth in rectangular and other sections which are symmetrical about their neutral axes. In such sections the flange stress f which causes the stretch setting up the initial gradient is measured in the outermost fibres. For such sections, therefore, the formula can be further simplified, for $Zd = \frac{I}{\frac{1}{2}d} \times d = 2I$, and $\frac{CWl^3}{EZd} = \frac{CWl^3}{2EI}$, and if the foregoing values of C be halved, a new constant $n (= \frac{1}{2}C)$ can be used with the formula $\triangle = \frac{nWl^3}{EI}$.

The values of n may be tabulated as follows :*—

	Uniform Section.	Uniform Flange Stress.
Cantilever concentrated load -	$\frac{1}{3}$	$\frac{1}{2}$
,, distributed load - -	$\frac{1}{8}$	$\frac{1}{4}$
Girder central load - - -	$\frac{1}{48}$	$\frac{1}{32}$
,, distributed load - -	$\frac{5}{384}$	$\frac{1}{64}$

---

* These values are, as stated, for parallel girders only. Variations in depth render the computation of n more intricate. In

For **T** and other sections which are not symmetrical about their neutral axes, the formula can be used in the same form. In the form of $\frac{\mathrm{CW}l^3}{\mathrm{EZ}d}$ the factor d would be twice the greatest distance (y) of the neutral axis from the outer fibres.

The fibre stress to which such a section would be calculated would be that in the extreme fibres at the tip of the web, and the gradient setting up deflection would be determined by the stretch of these fibres ÷ the distance Y. This gradient would be just the same as that determined by the lesser distortion in the fibres at the top of the table ÷ the lesser distance X. But as the lesser fibre stress in the table of a **T** section is not calculated, therefore the lesser distance X is not used. The deflection is, in fact, precisely the same as that of a symmetrical section of a depth equal to $2y$ under the same flange stress, and is therefore calculated on precisely the same way.

In the practical calculation of beams the flange stress f is almost invariably calculated first, and when this is known the factors of load, section modulus, and bending moment can be disregarded, the deflection being merely determined by the span and depth, or $\triangle = \frac{pfl^2}{\mathrm{E}d}$, p being a constant varying with the type of beam and method of loading. E for structural steel can be taken at 13,000 tons, and for a working stress of $7\frac{1}{2}$ tons per square inch, the formula can be simplified to

$$= \frac{pl^2}{d} \times \frac{7{\cdot}5}{13{,}000} = \frac{pl^2}{\mathrm{D}} \times 0.000576,$$

or if S in feet be substituted for l in inches—

$$= \frac{p\mathrm{S}^2}{d} \times 0.083.$$

---

cases where a uniform flange stress is secured by variations in depth, n may be taken at $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{24}$, and $\frac{1}{36}$ for the four cases given.

p can be tabulated as follows :—

Value of p in formula $\triangle = \frac{pS^2}{d} \times 0.083$.

	Uniform Section.	Uniform Flange Stress and Depth.
Cantilever concentrated load	$\frac{2}{3}$	1
,, distributed load	$\frac{1}{2}$	1
Girder concentrated load	$\frac{1}{6}$	$\frac{1}{4}$
,, distributed load	$\frac{5}{24}$	$\frac{1}{4}$

HANDY FORMULÆ FOR THE DEFLECTION OF WOODEN BEAMS.—If a rectangular beam of clean deal 1 in. broad and 1 in. deep be placed over a span of 1 ft., and loaded centrally with a weight of 1 lb., the deflection will be found to be in the neighbourhood of 0.0003 of an inch. Any increase in breadth would decrease the deflection directly, and any increase in depth would decrease it according to the cube of such increase in depth.

If the load be distributed by applying it in the shape of flexible strips of lead, the deflection for a 1-lb. load over a 1-ft. span would be about 0.0002, for a 2-ft. span 0.0016, and for a 3-ft. span 0.0054, etc., and any alteration of breadth and depth would have the same effect as in the case of the concentrated load.

It is possible to obtain very useful handy formulæ from the results of these experiments.

In everyday practice it is very frequently necessary to know for wood beams what is the load in cwt., either concentrated or distributed, which will cause a deflection not exceeding $\frac{1}{40}$ in. per foot of span or $\frac{1}{480}$—the deflection which is usually considered to be sufficient to crack plastered ceilings.

This limit of deflection for a 1-ft. span would be represented by $\frac{12}{480}$ S, or 0.021S. With a beam 1 in.

in breadth, 1 in. in depth, and 1 ft. in span a deflection of 0.002 was caused by a distributed load of 1 lb. Had the load been 112 lbs. the deflection would have been about 0.0224 in. In other words, for a unit span of 1 in feet, a unit breadth of 1 in inches, and a unit depth of 1 in inches, a unit distributed load of 1 in cwt. causes a deflection of approximately $\frac{1}{40}$ in. per foot of span.

For any beam of clean fir, therefore, it can be assumed that the distributed load in cwt. which will produce a deflection of $\frac{1}{40}$ in. per foot span will be expressed by $\dfrac{bd^3}{S^3}\begin{array}{l}\text{in inches.}\\ \text{in feet.}\end{array}$

For concentrated central loads the results should be multiplied by $\frac{2}{3}$, and for hard woods the deflection may be taken for practical purposes as the same as deal.

The above handy formula can be further extended. We have ascertained that the load in timber beams which causes a deflection of $\frac{1}{40}$ in. per foot of span can be shortly put as $W = \dfrac{bd^3}{S^3}$, and we can similarly use the results of experiments to determine the deflection of a timber beam under any load. Our model beam deflected .03 or $\frac{1}{30}$ in. under a concentrated load of 1 cwt. in a span of 1 ft.; and .02 under a distributed load. Obviously, therefore, to obtain the deflection in any beam we have merely to regard the conditions, *i.e.*, breadth and depth in inches and span in feet, of our beam in the experiment as unity, and multiply by the relation $\dfrac{S^3}{bd^3}$, the deflections of $\frac{1}{30}$ in. per cwt. of load for beams under concentrated load, and $\frac{1}{50}$ in. for distributed load.

If instead of beams we had cantilevers, we could easily use the same experimental results and increase the figure of $\frac{1}{30}$ in. by the relation between the deflections of girders and cantilevers shown in the table. Thus the deflection of a cantilever 1 in. × 1 in. × 1 ft. span, under a concentrated load of 1 cwt., would be to the deflection $\frac{1}{30}$ in. for a beam of the same section and load in the proportion of $\frac{2}{3}$ to $\frac{1}{24}$, or would be—

$$\frac{1}{30} \text{ in.} \times \frac{2}{3} \div \frac{1}{24} \text{ in., or } \frac{1}{30} \times \frac{2}{3} \times \frac{24}{1} = \frac{48}{90} \text{ in., say } \frac{1}{2} \text{ in.}$$

It is, however, in practice more convenient to know what deflection is going to take place in a timber beam loaded to the extent of the handy formula for timber beams: Safe distributed load in cwt. $= \dfrac{bd^2 \text{ in inches}}{\text{S in feet}}$. Under these circumstances, assuming the modulus of elasticity for large scantlings to be 9,000 cwt. per square inch and f to be 9 cwt. per square inch, which is the safe f assumed in that formula, then—

$$\triangle = \frac{5}{24}\frac{fL^2}{ED}. \quad (\text{L in inches} = \text{S}12 \text{ in feet.})$$

$$= \frac{5 \times 9 \times 12\text{S} \times 12\text{S}}{24 \times 9000 \times d} = \frac{\text{S}^2}{30d}.$$

So that for wood beams loaded with the safe distributed weight allowed by the handy formula (or any other arrangement of weight involving a flange stress of 9 cwt. per square inch) the deflection can be calculated from the very simple formula $= \dfrac{\text{S}^2}{30d}$. If a modulus of 10,800 be assumed, the formula becomes $\dfrac{\text{S}^2}{40d}$.

Limiting Ratio of Depth to Span for given Ratios of Deflection to Span.—It is often convenient in practical design to determine the limiting proportion of depth to span which will keep deflection within defined limits. For fir timber beams and joists under distributed loads the limit of deflection usually fixed is $\frac{1}{40}$ in. per foot of span or $\frac{1}{480}$, and the material is almost invariably calculated for an f of 9 cwt. per square inch. Assuming a modulus of 10,800 cwt., the deflection in such beams—

$$\triangle = \frac{5}{24}\frac{fl^2}{Ed}, \text{ and when this is } \frac{1}{480}l,$$

then
$$\frac{l}{480} = \frac{5 \times 24 \times 9}{24 \times 10,800} \times \frac{l^2}{d},$$

or
$$\frac{l}{d} = \frac{10,800 \times 24}{480 \times 5 \times 9} = 12,$$

or depth in inches = span in feet, which is obviously much more than experience has found necessary.

In this connection it should be recollected that ceilings are cracked not by the deflection due to the *total* load, but by that portion of it which is represented by the superimposed load. The deflection due to structural load is constant and permanent from the time the structure is completed. When the superimposed load is about half the total load, the old rule for floor joists, depth in inches = half span in feet + 1, would be correct.

For rolled joists or steel girders as beams under distributed load with uniform flanges working to a stress (f) of $7\frac{1}{2}$ tons per square inch and using steel possessing a modulus of 13,000 tons

$$= \frac{5}{24} \times \frac{fl^2}{Ed}, \text{ and when this is } \frac{l}{480},$$

then
$$\frac{l}{480} = \frac{5 \times 7{\cdot}5}{24 \times 13000} \times \frac{l^2}{d},$$

$$\frac{l}{d} = \frac{24 \times 13{,}000}{480 \times 5 \times 7{\cdot}5} = 17{\cdot}5.$$

The limiting deflection of rolled joists is more frequently taken at about $\frac{1}{360}$, under which conditions the ratio $\frac{l}{d}$ would be about 24, or *depth in inches = half span in feet.*

EFFECT OF END FIXING ON DEFLECTION OF CANTILEVERS.—Before leaving the subject of deflection it is necessary to note one practical point with regard to cantilevers. It will be found that these often show a deflection greater than is expected. This is because the formula assumes that one end of the cantilever is fixed ; but absolutely fixed ends are in ordinary building work almost impossible, and a very slight yielding of the material anchoring the short arm may mean a considerable drooping of the end of the long arm. Take the case of a cantilever 12 ft. long built into a wall with plates top and bottom, and so arranged that the effective centre of the top plate carrying the counterweighting masonry is 18 in. from the front edge of the bottom plate upon which the long arm would tend to rotate. If the top plate moved upwards under the action of the load, and compressed the counterweighting of masonry to the extent of $\frac{1}{16}$ in., the long arm would drop eight times as much, or $\frac{1}{2}$ in., in addition to the elastic deflection. In the case of a cantilever which is a prolongation of a girder over one of the bearings, the fixing of the cantilever is obviously flexible. In the case of a cantilever loaded with a distributed weight and counterweighted by means of such a girder similarly loaded and of the same span as in Fig. 110, the deflection is just double

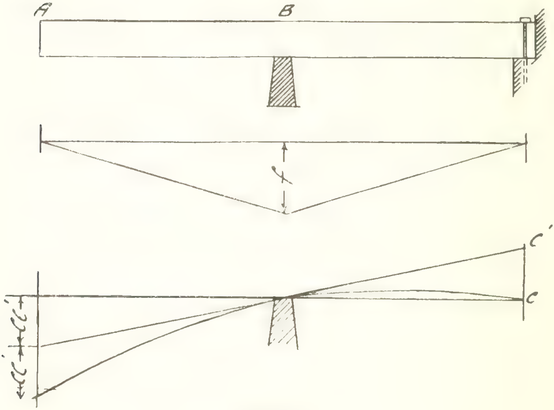


FIG. 110.

the normal, the difference being due to the flexibility of the counterweighting arm.

Cases necessitating the exact calculation of the deflection of balanced cantilevers do not often arise in ordinary building practice, cantilever arms being usually kept short, but, if necessary, simple methods of calculating them will be found in Professor T. Claxton Fidler's well-known book on Bridge Construction,* to which the author is indebted for the foregoing simple diagrammatic treatment of the theory of deflection.

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* "A Practical Treatise on Bridge Construction," by T. Claxton Fidler, M.I.C.E., published by Charles Griffin & Co. Ltd.

PART III

WALLS AND FOUNDATIONS

CHAPTER XVII

RETAINING WALLS AND BUTTRESSES

Failure of Walls and Buttresses Subject to Lateral Thrust—Failure by Overturning—Failure by Sliding—Critical Angle and Angle of Repose—Inclined Resultant Pressure—Coefficient of Friction—Friction Independent of the Area of Surface in Contact—Angle of Repose—Angles of Repose of Different Materials—Factor of Safety against Sliding—Limits to Batter of Retaining Walls.

FAILURE OF WALLS AND BUTTRESSES SUBJECT TO LATERAL THRUST.—Walls, buttresses, and chimney stacks, subject to lateral thrust or wind pressure, can fail in any one of three principal ways, viz. :—

1. By overturning bodily or at any point.
2. By sliding on the foundations.
3. By concentration of pressure in the neighbourhood of the toe of the foundation, causing either failure of the foundation, breaking of the footings under transverse stress, or crushing of the material of the wall itself.

FAILURE BY OVERTURNING.—The simple experiment described on page 50 of a brick balanced on a lever first on its 9-in. side, then on its $4\frac{1}{2}$-in. and 3-in. sides, and finally hung on a string, proved that collections of parallel forces, such as the weights of different particles of a body, may be considered, for purposes of leverage, as acting at their centres of gravity.

Taking the simple case of a chimney stack acted upon by horizontal wind pressure shown in Fig. 111,

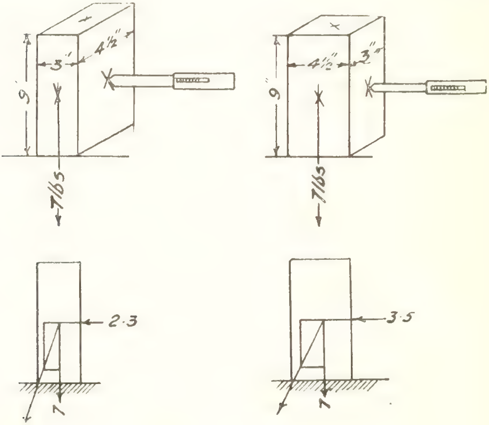


Fig. 111.

the opposing forces for and against overturning are on the one hand the wind, and on the other the weight of the stack. The wind may be considered as acting equally on all parts of one face of the stack, and therefore for purposes of leverage it acts as if concentrated on the centre of the face.

The stack can be represented by a brick standing on edge, the brick weighing 7 lbs., and its base measuring 3 in. in the direction of the wind pressure.

The concentrated load can be represented by a delicate pointed compression balance. The centre of gravity of the brick (determined by balancing it in a string loop) should be marked on the surface, and it will be found that a push of 2.3 lbs. is necessary to overturn the brick. If the brick be turned so that its base in the direction of the wind pressure measures $4\frac{1}{2}$ in.—the push registered is $3\frac{1}{2}$ lbs. It is also found that a lesser push is necessary above the level of the centre of gravity, and a greater push is necessary below it; and that if the push be applied at a point sufficiently low, the brick no longer tends to overturn, but to slide forward.

On a particular brick tested when standing on a wood drawing-board, it was found that sliding took place when pressure was applied at a point 3 in. above a base of 3 in. wide, the pressure being $3\frac{1}{2}$ lbs. Sliding also took place when the drawing-board was tipped up to an angle of 27° with the horizontal.

When the same brick was placed upon another brick instead of upon a smooth board, the point where lateral pressure caused sliding instead of overturning was decidedly lower, viz., 2.1 in. above the base, the pressure required being 5 lbs.; and the bricks had to be tipped up to an angle of 35° before they slid over each other.

When the push was distributed all over the surface of the brick by means of a piece of light board, the same results were obtained.

Consider first the overturning moments involved, Fig. 111. The pressure acts at a leverage of $\frac{h}{2}$, or $4\frac{1}{2}$ in., and it is tending to overturn the brick or stack on the further bottom edge in an anti-clockwise direction. The weight of the brick being all concentrated on the right-hand side of the point of rotation, is obviously in a position to oppose the tendency towards overturning by pulling downwards in a clockwise direction, with a leverage about the point of overturning of *half* the width of the base, viz., the distance between a vertical line through the centre of gravity and the point of rotation. As usual, therefore, a condition of equilibrium is obtained, represented by a simple equation of four terms :—

For overturning by lateral pressure applied at $\frac{H}{2}$. Overturning moment = moment of resistance.

$$P \times \frac{H}{2} = W \times \frac{B}{2},\ P = W\frac{B}{H}.$$

If $W = 7$ lbs., and $B = 3$ in., and $H = 9$ in., then

$$P = 7 \times \frac{3}{9} = 2.3 \text{ lbs.}$$

When $W = 7$ lbs., $B = 4\frac{1}{2}$ in., and $H = 9$ in., then

$$P = 7 \times \frac{4.5}{9} = 3.5 \text{ lbs.}$$

If any three of the four terms are known, the fourth can be deduced. If the height and weight and base of a stack be known, the wind pressure which will overturn it can be calculated. If a certain wind pressure be fixed upon as being the maximum pressure to which any stack can be subjected, it is easy to ascertain to what height a stack of given section can be carried, or what cross section a stack of a given height must possess.

Similarly if a horizontal thrust as from an arch be exerted against a wall at a certain height, it can be ascertained whether the wall will require buttresses, and if so, of what weight and base (in other words, of what section), in order to be safe against overturning. If, on the other hand, a wall or buttress of a certain section be either in existence or necessitated by other considerations, it can be seen whether it would or would not safely resist without overturning a given thrust at any height, or whether such a thrust must be neutralised by tie-rods.

FAILURE BY SLIDING.—Returning to the case of the experimental brick. Its weight being 7 lbs., and its base 3 in. wide, its moment of resistance against overturning would be, therefore, 7 lbs. × half its base, or 10½ in.-lbs. When standing on smooth wood, the height at which a lateral pressure just tended to cause sliding in preference to overturning was 3 in., the extent of that pressure being 10½ in.-lbs. ÷ 3 in., or 3½ lbs., as was found. When standing upon another brick, the 5 lbs. applied at 2.1 in. above the base would have exercised an overturning moment of 10.5 in.-lbs., which was again exactly equal to the moment of stability.

Draw a vertical cross section of the brick and indicate by vertical and horizontal lines on it the forces for and against overturning, viz., the push acting horizontally and the weight acting vertically through the centre of gravity, as in Fig. 111, and measure off on these lines lengths corresponding to the forces to any convenient scale. Complete a parallelogram and its diagonal will represent the inclined thrust on the foundation.

It will be found that when the forces are such that overturning takes place, the diagonal thrust extended would always fall just outside the base.

CRITICAL ANGLE AND ANGLE OF REPOSE.—When a number of careful measurements are made of the conditions which cause sliding instead of overturning, and similar diagrams are drawn of each experiment, it is found that, for any given materials of wall and base, sliding invariably takes place when the inclined thrust (viz., the resultant of the thrust and the weight of the wall) cuts the base at, or at less than, a certain angle. With different materials this "critical" angle varies, but with the same two materials it is invariably the same, no matter how heavy or how light the specimen may be, nor how large or small may be its base. It depends upon the *materials* and their state, and not upon their weight or dimensions. If a specimen of one material be laid upon a slab of another material, and the slab be tipped up until the material slips, a similar critical *angle of tip* will be found; and with two given materials sliding invariably takes place at the same angle of tip, whatever the size or weight of the pieces experimented on. A particular brick slides down a wood drawing-board at a tip of 27° —it slides down another brick of the same make at 35°, and so on. As will be seen from Fig. 112, increasing the angle of tip has the effect of *reducing* the angle between the vertical direction, in which gravity acts upon the weight of the specimen and the surface of the slab. This angle of tip is, in fact, the complement of the critical angle of friction and is called the angle of repose.

The following simple experiments will be found somewhat instructive. Press a walking stick against the ground and see at what angle it slips. Increase the pressure and see whether the angle is varied.

Lay a brick on a flat wooden board, first on its $4\frac{1}{2}$-in. face, then on its 9-in. face with a heavy weight on the top, then on its 3-in. face, loaded or unloaded, and see what difference, if any, the weight and area of base makes to the critical angle of sliding.

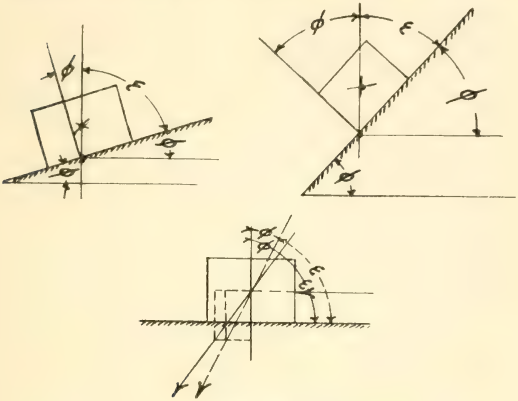


FIG. 112.

INCLINED RESULTANT PRESSURE.—So far as simple overturning is concerned, there is no need to draw a parallelogram of forces, but for the purpose of ascertaining whether the wall is liable to slide or to fall by excessive pressure at the toe, it is necessary to know the direction and amount of the inclined resultant.

It is important to fully understand what this resultant force really is. If two forces tend to move a substance or structure in two directions, that substance or structure will tend to move in *either* of the two directions desired by the forces, *or* in a direction somewhere in between the two. Take the forces acting on the pin of a bell-crank lever with an 18-in.

horizontal and a 6-in. vertical arm, Fig. 113. If the end of the horizontal arm were loaded with, say, 3 lbs., and balanced by a pull of 9 lbs. on the vertical arm, then the pin at the intersection of the long and short arms would move downwards *if it could* with a force of 3 lbs. If it could not move downwards but could move in a horizontal direction, it would do so with a force of 9 lbs. If it could move neither vertically nor horizontally, then both forces would combine to push it in the direction of the resultant: and the amount and direction of this resultant is given by the length and direction of the diagonal of the parallelogram, whose sides equal the two forces in amount and direction, the direction of the resultant depending upon the relation between the two forces which call it into being. If the vertical force is great in proportion to the horizontal, the resultant is nearly vertical. If the horizontal force is in excess the direction of the resultant is flatter. If they are equal it is at an angle of 45°. It is important to recollect that the pin of the bell-crank lever would tend to move horizontally with a force equal to the horizontal pull on the lever *if it were able to do so;* and a wall subjected to lateral thrust has a similar tendency to slide forward.

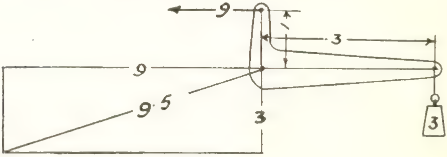


FIG. 113.

It is restrained from doing so by friction, in the same way that the lever pin is restrained by its bearings, and when the horizontal force is great enough to overcome this restraint, the wall will and often does slide. Overturning only takes place when sliding is impossible—the first effort of a lateral thrust is to cause sliding.

When the attempt is made to overturn a brick, it is found that a slight lateral push near the top causes overturning, and that as the push is applied lower and lower, the force necessary to create an overturning moment greater than the moment of stability of the brick becomes greater and greater as its advantage of leverage decreases. At last a point is reached when the horizontal thrust necessary to cause overturning is equal to the friction between the brick and the table, and then apparently the brick does not care very much whether it slides or overturns. When the lateral force is applied the least trace lower, then the force necessary to cause overturning is greater than that necessary to overcome friction, and failure occurs (as it always does occur) along the path of least resistance, and sliding takes place instead of overturning.

When the bed joint is made of two bricks, the friction is naturally greater than before and resists longer the tendency to slide. Of course, when the horizontal force is less than the resistance of the brick to sliding or overturning, failure does not take place by either method.

COEFFICIENT OF FRICTION.—As the direction of the resultant of any two vertical and horizontal forces is governed by their relative amounts, it follows that the critical angle for any material can be expressed in terms of vertical weight and horizontal thrust. In other words, that a given weight of any given material on a horizontal base will always slide under the action of a lateral force amounting to a certain *proportion* of that weight. *This proportion is called the coefficient of friction for that material.*

A 7-lb. brick resting on smooth wood was found to slide when subjected to a lateral force of $3\frac{1}{2}$ lbs., or 0.5 of its weight. The resultant under these conditions was inclined at an angle of 63° with the surfaces in contact. The same conditions would obtain if the board were tipped up to an angle of 27°, and when this was done sliding similarly took place.

The least critical angle for average dry brick on dry brick is about 55°, which is the direction of the resultant of a horizontal force of 0.7, and a vertical force of 1. Therefore, 1 ton of brickwork might be expected to slide on uncemented joints under a lateral thrust of anything over 0.7 ton. With "green" mortar joints a push of 0.47 of a ton would suffice to cause sliding. Similarly, concrete will slide on dry clay at an angle of 27°, *i.e.*, if exposed to a lateral force of half its own weight; and on wet clay at 18°, or if exposed to a thrust equal to one-third of its own weight.

FRICTION INDEPENDENT OF THE AREA OF SURFACES IN CONTACT. — Whenever the lateral thrust is equal to or greater than the weight × the coefficient of friction, sliding will take place. It is necessary to carefully observe that the size of the base makes no difference whatever to the critical angle. If on a tilted board similar weights be placed on two pine slips, one of which is half the size of the other, they start sliding at exactly the same angle of tip; and if measurements be made of the horizontal force necessary to slide them along a horizontal surface, it would be found that the same force is necessary whatever the size of the surfaces in contact.

The fact that *friction is independent of the area of surface in contact* is surprising, but there can be no doubt of it if one considers the enormous forces acting between the driving wheels of a locomotive and the rail surface—the area of surfaces in contact is practically nothing—they are to all intents and purposes points.

Angle of Repose.—It is a matter of common observation that when various kinds of earth, gravel, etc., are heaped up, the sides tumble more or less immediately to a certain angle (ϕ, Fig. 114)—gravel stands at one angle, earth at another, broken stone at another.

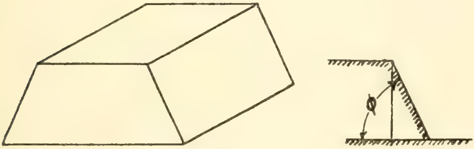


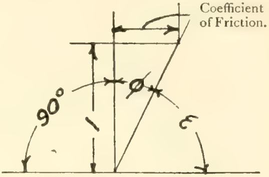
Fig. 114.

This angle is called the *angle of repose* of that material. It is identical with the angle of tip at which specimens of the material will slide on each other, and it is the complement of the critical angle of friction (see Fig. 115). The earth thrust on retaining walls cannot be calculated without a knowledge of angles of repose of different soils, and even if a wall were safe from any danger of sliding on its bed joints or foundations, or the foundations upon the earth, it must be remembered that even though there may be no danger of the concrete foundations sliding on the earth, yet immediately below the concrete the earth might slide *on itself* if the inclined thrust from the concrete strike it at an angle flatter than the angle of repose of the earth upon itself. The following are a few angles of repose of different materials—a fuller list may be found in almost any text-book.

ANGLES OF REPOSE OF DIFFERENT MATERIALS.

	Angle of Repose with the Horizon. ϕ	Coefficient of Friction ($\tan \phi$).
Masonry on brickwork, dry - -	31-35°	.600-.700
„ „ new wet mortar - - - - - -	25°	.466
Masonry on brickwork, damp mortar	36°	.726
„ on dry clay - - - -	27°	.509
„ on moist clay -	18°	.325
Wood on stone - - -	22°	.404
Iron on stone - - - - -	17-35°	.306-.700
Wood on wood laid dry - -	14-26°	.249-.488
Water - - - - - - -	0	Zero
Mud, semi-liquid - - -	5°	.087
Sand, fine, dry - - - - -	32°	.625
„ wet - - - - - -	26°	.488
Earth, ordinary - - - - -	14-45°	.249-1.00
„ colsolidated, dry - -	45°	1.00
Clay, wet, recently dug - -	15-25°	.288-.466
„ dry, well drained - -	26°	.488
Gravel, compact - - - -	40°	.816
„ with sand - - - -	32°	.625
Shingle, loose - - - - -	36°	.703
Rubble stone - - - - -	45°	1.000
Peat, wet - - - - - -	14°	.249
„ dry, consolidated - -	45°	1.000

The coefficient of friction corresponding to any angle of repose can be found by setting up the angle on a vertical line, as in Fig. 115, cutting off a length of the vertical line equal to 1, and measuring the horizontal distance between the vertical and the inclined lines. In other words, it is the trigonometrical tangent of the angle of repose ϕ, or the cotangent of the critical angle ϵ.



ϵ = *Critical Angle of Friction.*
ϕ = *Angle of Repose.*

FIG. 115.

FACTOR OF SAFETY AGAINST SLIDING.—Although it is not customary to run much risk as far as sliding is concerned, yet the necessary margin of safety found by experience is quite a low one, viz., $\frac{1}{5}$. This must not be confounded with a Factor of Safety of 5, and is better written 0.2, *i.e.*, we can (if *certain* of our coefficients) work up to within $\frac{4}{5}$ of the danger point. Thus if a horizontal force of 50 would cause a structure to slide, it could be trusted to resist the sliding tendency of a horizontal force not exceeding 40.

CHAPTER XVIII

RETAINING WALLS AND BUTTRESSES (*continued*)

Eccentric Pressure on Joints and Foundations—Maximum and Minimum Pressures.

ECCENTRIC PRESSURE ON JOINTS AND FOUNDATIONS.—The effect of eccentric pressure on bed joints and foundations is a subject of importance. First, because crushing in the neighbourhood of the toe or outer edge of the bed joint is the most frequent cause of failure of walls, arch voussoirs, and piers; and secondly, because if the effect of eccentric or inclined pressure on a bed joint be clearly grasped, the problem of eccentric loading of piers, stancheons, and columns, and incidentally the theory of the strength of columns, becomes comparatively simple. Eccentric loading is constantly occurring in practice; in fact nearly every column or stancheon ought to be treated as loaded out of the centre. Unfortunately, eccentric loading is one of those subjects which mathematicians delight to clothe in mysterious formulæ, but if viewed in the light of common-sense there is no reason why it should be any more complicated than the mathematics of a simple lever.

The effect of applying lateral thrust to a wall which cannot slide forward is to shift the position of the centre of pressure on the foundation from that due to the weight of the wall alone to that of the resultant of the weight of the wall and the lateral thrust, so that the centre of pressure on the foundation is no longer in the centre of the base, and the same thing occurs at every bed joint below the point of application of the lateral thrust.

Imagine a retaining wall for earth being built. A vertical load is gradually built upon the earth foundation. Then when the earth backing is filled in, the wall is subjected to a lateral thrust from the earth which it is designed to retain. There is now the weight of the wall and the lateral thrust, and the combined effect is an inclined resultant, ready to split up again into its constituent forces wherever it finds a path along which either one of them can cause motion. This inclined resultant force can be imagined as travelling downwards and looking out sideways and downwards for a chance of exercising either of its two constituent faculties, viz., downward pressure or lateral sliding. If it found on its way down to the base a smooth joint, it would exercise its lateral component and cause sliding along that joint. If it found a soft place it would crush it downwards by means of its vertical component. It must be remembered that if there is so much friction on the joints and on the foundation that the inclined force finds that it cannot cause lateral or sideways movement, then it causes downward pressure, and the downward pressure which it can exert is the vertical component of itself at the point in question. Suppose the inclined force to have reached the base without having been able to do any mischief, and finds that the base possesses too much friction to permit of sliding, then the horizontal component of the inclined force being successfully resisted by friction, the inclined force acts through its vertical component, and the base need only be considered as subjected to a vertical force at the point where the inclined resultant cuts it. Immediately below the base, however, *both* forces must be provided for again.

We have been regarding the forces in the wall as being concentrated in straight lines, because for purposes of leverage parallel forces may be considered to act as if concentrated at their centre of gravity; and it is easier to think of single forces acting along lines. It is easier to think of the pressure from the tip of one's finger than it is to realise the varying pressures from the palm of one's hand—one is a single pressure, the other is made up of varying pressures distributed over a surface. But it must not be forgotten that in a wall or on a section it is really *varying pressures on a surface* which are being dealt with. Their effect *for purposes of leverage* can, without error, be expressed as concentrated into single forces acting at the common centres of gravity; just as the muscular force of the hand can be concentrated into one finger tip for purposes of leverage, but *not* if it is desired to know what happens to the surface or section under pressure.

In order to appreciate how lateral pressure on a wall is transformed into excess of pressure at the toe or tension behind the toe, it is useful to recall yet once more what takes place in a bell-crank lever. It will be recollected that the forces required to keep it in equilibrium are proportional to the lengths of the arms. When the arms are equal a weight of 1 lb. hung on the horizontal arm could be balanced by a pull of 1 lb. on the vertical arm—when the horizontal arm is three times as long as the vertical a 1-lb. weight hung on the end of the horizontal arm requires a pull of 3 lbs. on the end of the vertical. If, therefore, a parallelogram be drawn of which the length of any two adjacent sides represent the respective lengths of the lever arms, then the length of the two remaining sides would represent the proportions between the weights. A parallelogram having two adjacent sides equal to 3 and 1 representing the weights would have two remaining sides of 1 and 3 representing the leverages. Either weight multiplied by its leverage would represent the clockwise or anti-clockwise turning moment of that weight, and would be represented by the area of the parallelogram, 3 lbs. × 1 ft. = 3 ft. × 1 lb.

Now let a short length of wall be considered as a cantilever standing up on end. The moment of the horizontal force around any point in the base would be its amount × its height above the base, in precisely the same way that the bending moment of a load on a cantilever at the support is the load × its horizontal distance from the support. Now draw, as in Fig. 116, a parallelogram of forces to such a scale that the weight of the wall is represented (to that scale) by a line of the same length as the leverage A B of the lateral force. This is not difficult. Suppose the weight of the wall to be 3 tons and the leverage 10 ft. and the wall had been drawn to a scale of 1 in. to the foot, then the scale to use for force would be 1 ton $= \frac{10 \text{ in.}}{3}$. If now the length A D be drawn equal to the lateral force to this scale and the parallelogram be completed, then its resultant will cut the base at C.

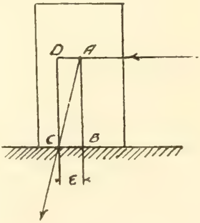


FIG. 116.

It is now obvious that the vertical weight × the length C B equals the bending moment caused by the lateral force × its leverage. This is a most important result, for it gives the key to all problems of eccentric loading. *The B.M. of a lateral force A D acting on a wall or column = A D × its leverage B A, which also equals the vertical component D C of the inclined thrust × its eccentricity C B.*

MAXIMUM AND MINIMUM PRESSURES.—To ascertain the maximum and minimum pressure on a horizontal joint, the vertical component of all the forces above that joint should first be considered as acting vertically downwards. Then if that vertical component is for any reason acting at a point which is not the centre of gravity of the horizontal surface of the bed joint, there occurs *in addition* a bending moment, which increases the original pressure on one side of the centre of gravity, and decreases it in the other; and that bending moment in the case of a wall subject to lateral thrust can be calculated *either* by multiplying the lateral thrust by its leverage, *or* by multiplying the vertical component by the extent of its eccentricity. In the case of beams and cantilevers the extreme flange stress can be obtained, as explained in Chapter XII., by dividing the bending moment by the section modulus (Z), which in all rectangular sections is $\frac{bd^2}{6}$. A short length of wall may be considered as a vertical cantilever of rectangular section of which the Z is $\frac{bd^2}{6}$, and the extreme stress in such a wall is obtained by dividing the B.M. of the lateral force or the eccentric load by $\frac{bd^2}{6}$. But the B.M. does not act alone. There was a pressure over the whole base before the B.M. arrived, and all the latter can do is to increase this pressure at the toe and decrease it at the back. To find the total pressure at the toe, the following simple expression is used :—

$$\text{Maximum pressure at toe} = \frac{W}{A} + \frac{W \times e}{\frac{bd^2}{6}}.$$

$$\text{Minimum pressure at toe} = \frac{W}{A} - \frac{W \times e}{\frac{bd^2}{6}}.$$

When W = the vertical component.
e = its eccentricity.
A = the area of the section.

When $\frac{W \times e}{\frac{bd^2}{6}}$ is greater than $\frac{W}{A}$, then there is tension at the back of the wall.

Fig. 117, *a* to *d*, show examples of how this pressure may be distributed. It will be seen at once that when a force above the base calls into play reactions in the

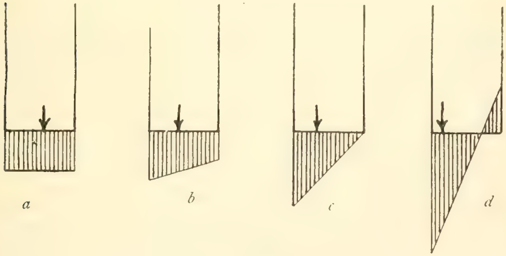


FIG. 117.

base, then the centre of gravity of these reactions must be under the centre of gravity of the generating force. When the minimum force is 0 and the diagram of reactions is triangular, the centre of gravity of the reactions must be at one-third the width of the base from the toe ; so that if tension in the joint is to be avoided, the resultant of the weight and lateral force must be kept within the middle third of the base.

There has been much discussion as to what really takes place in walls whose joints afford no tensile strength, and which depend for stability solely upon the weight of the wall. The usually accepted theory is that the resultant pressure creates a triangle of reactions under it whose centre of gravity is directly under the resultant—in other words, that the weight of the wall is concentrated on a space having a width equal to three times the distance of the resultant from the toe, the rest of the base joint simply tending to open. If the base be regarded as rigid it is difficult to grasp this, and, on account of that mental difficulty, various complicated theories have been put forward. But neither masonry, brickwork, nor concrete *are* rigid materials, on the contrary, they are highly elastic within narrow ranges. If the effect of eccentric pressure be tried on two pieces of indiarubber placed together, there will be difficulty in mentally digesting the usually accepted theory.

CHAPTER XIX

RETAINING WALLS AND BUTTRESSES (*continued*)

Various Types of Retaining Walls—Earth Pressure at Back of Retaining Walls—Retaining Walls only Support Half the Wedge of Earth Retained above the Angle of Repose—Surcharged Walls—Retaining Walls for Water—The Design of Retaining Walls—Typical Sections of Retaining Walls—Weight of Buildings near the Back of Retaining Walls.

VARIOUS TYPES OF RETAINING WALLS.—The usual method of retaining the wedge of earth between the vertical and natural slope is by building what is known as a gravity retaining wall of brick or concrete, too heavy to be overturned or pushed forward, and too strong to be crushed at the toe. Instead of the whole wall being made heavy enough to resist safely the lateral thrust of the earth wedge, the necessary weight may be concentrated in buttresses, the intervening spaces being spanned by thinner walls, arched on plan from buttress to buttress, as in Fig. 118.

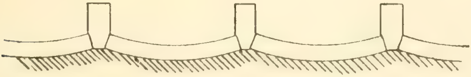


FIG. 118.

Another method is to call into play part of the weight of that portion of the earth which lies below the angle of repose, either by anchoring the wall back into it, as in Fig. 119, or by constructing a thin

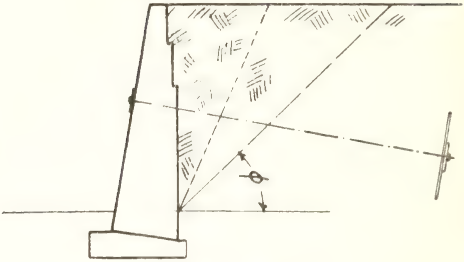


FIG. 119.

reinforced concrete breast wall firmly attached to a broad base, as in Fig. 120.

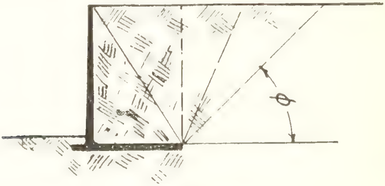


FIG. 120.

Ordinary building practice is generally confined to walls that depend upon their own weight, but the principles which govern all other kinds of wall are precisely the same.

EARTH PRESSURE AT BACK OF EARTH RETAINING WALLS.—Although every kind of soil has its own distinctive and particular angle of repose or slope, which the sides of a heap of such soil tend to assume, yet freshly excavated earth will generally stand at first at a slope which is considerably steeper. For instance, the sides of an excavation in clay may remain vertical for some time at quite considerable depths; but under the action of weather the exposed surfaces crumble and slip until the soil stands at its natural angle of about 30° from the horizontal. This property of freshly excavated soil is sometimes utilised to save the cost of heavy retaining walls, by substituting thin breast walls which will protect it from disintegrating influences. But this can only be done safely when the designer has extensive experience of the soil and locality. The risk of slips arising from fissures and from water in the ground behind the wall is considerable, and the practice is of doubtful economy, and not to be recommended.

The object of all forms of earth retaining walls is to prevent the wedge of earth *between the vertical and the natural* angle of repose from tumbling forward. The lateral pressure which such a wedge exerts must first be ascertained.

If a heavy wedge of wood be placed on a plane which is inclined at an angle exceeding the complement of the limiting angle of friction, and so arranged that the wedge has one face vertical and one side horizontal, as in Fig. 121, it will be found that the

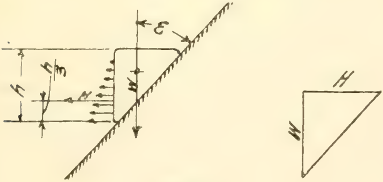


FIG. 121.

centre of horizontal pressure on the vertical face is at one-third of the height measured from the lower edge. The reason of this is obvious—the pressures due to the weight of the wedge increase in proportion to the depth, and the diagram of pressures is therefore a triangle, the centre of gravity of which is necessarily at one-third of its altitude. Also, it will be found that the amount of horizontal pressure is such that if a triangle be drawn whose angles are equal to those of the wedge, and with one vertical side (W) of a length representing the weight of the wedge, as in Fig. 121, then the horizontal side (H) of the triangle would represent the horizontal pressure. If now the vertical face of the wedge be considered as bearing against a smooth vertical plane, the action of the wedge would be to exert a horizontal pressure in such a plane which varies from top to bottom, and the total extent of that pressure would be the weight of the wedge multiplied by the proportion between its horizontal and vertical sides, whilst the centre of gravity, or the point at which it may be considered as acting, would be at one-third of the height; and this is the action of the wedge of earth retained on the back of a smooth retaining wall with a vertical back. But the backs of retaining walls are *not* smooth, and sometimes they are stepped in offsets. The constant tendency of the wedge is to slide downwards along the inclined plane, and if the vertical plane is sufficiently rough (as in the case of retaining walls with stepped backs) the horizontal lateral thrust will be deflected more or less downwards, and the bending moment on the wall would not be so severe. What actually takes place is still somewhat doubtful, but the usually accepted theory, which experience proves to be safe, is that of Dr Scheffler, which is that whilst the centre of the thrust occurs at one-third of the height, it is not the horizontal thrust of a wedge on a smooth vertical plane, but such a component of a horizontal thrust as would act *parallel to the angle of repose*, and which is, therefore, greater than a horizontal thrust, but is acting in a direction more favourable to stability. So that if at one-third the height a horizontal line be drawn representing the horizontal thrust of a wedge, and from the further extremity of that line a vertical line be raised, and from the other extremity a line be drawn parallel to the angle of repose, as in Fig. 122, the length of inclined line intercepted will give the direction and amount of the true thrust. Those who understand trigonometry will see that the same result will be obtained by multiplying the horizontal thrust of the wedge by the secant of the angle of repose. If the shape of the wedge of retained earth and its weight per cubic foot be known, it is easy to obtain the horizontal thrust, and the inclined thrust which it will exert on a rough wall can be ascertained graphically or by trigonometry.

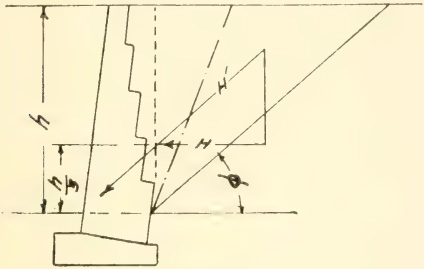


FIG. 122.

But up till now one curious and important phenomenon has not been considered. The total wedge of earth supported is, of course, the wedge lying between the angle of repose and the vertical, but owing to the friction of the earth particles such a wedge partly supports itself.

RETAINING WALLS ONLY SUPPORT HALF THE WEDGE OF EARTH RETAINED ABOVE THE ANGLE OF REPOSE.—If a box with one glass side and one end hinged at the bottom be filled with earth and the hinged end be slightly opened, it would be found that the earth would first tend to break away along a line *half-way between* the vertical and the natural angle of repose, and this half-wedge is all that a retaining wall is really called upon to support.

SURCHARGED WALLS.—If the surface of ground to be retained slopes upwards from the back of the wall as Fig. 123, the wall is said to be "surcharged,"

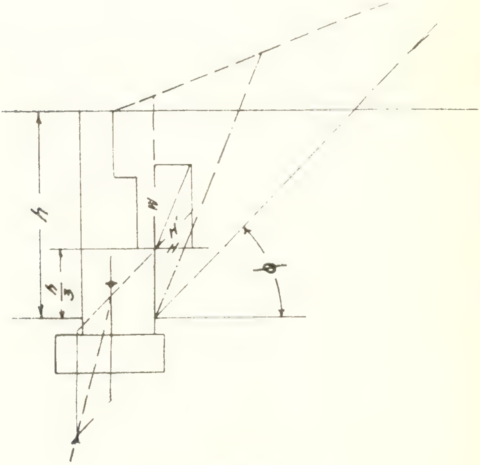


FIG. 123.

and in that case, to find the wedge supported it is necessary to continue upwards the line defining the lower limit of the half-wedge until it cuts the ground surface.

When a retaining wall has offsets at the back the earth above the offset is regarded as being, not part of the wedge supported, but part of the supporting wall; and is compounded with the weight of the masonry of the wall to find the true common centre of gravity. The effect of the earth lying on the offsets being to add a little to the effective weight of the wall and to shift its centre of gravity back to a somewhat more favourable position.

RETAINING WALLS FOR WATER.—The pressure which water exerts upon any point on the back of a retaining wall is always at right angles to the back of the wall, and is equal to the weight of fluid above that point. Thus the pressure upon a foot super. of the back of a wall retaining water at 10 ft. below the surface is the weight of a column of water 1 ft. × 1 ft. × 10 ft., or 62.5 lbs. × 10. The pressure upon any given unit of area of wall surface is quite independent of the amount of water retained—it is the same at the same depth whether it be in a pipe of 1 sq. ft. in sectional area or from a lake of a square mile or a hundred square miles.

THE DESIGN OF RETAINING WALLS.—The following are the complete steps in the design of an earth-retaining wall of any height.

The necessary particulars which must be ascertained or which must be assumed are:—

The nature of the ground supported—its angle of repose, its weight per cubic foot, the safe pressure which it will support as a foundation, and the weight per cubic foot of the material of which it is proposed to build the wall.

The angle of repose of the earth is then drawn from the base of a wall on a section of the earth to be supported, and the angle between it and the vertical is bisected to give the profile in section of the true wedge of earth which will act. This wedge and the wall are supposed for purposes of calculation to be 1 ft. in length. The weight of the wedge is calculated, and its horizontal thrust at one-third the height is ascertained either graphically or by means of the formula $\frac{1}{2}wh^2\tan^2\left(\frac{90-\phi}{2}\right)$.

Where w = weight of earth per cubic foot.

h = height of wall.

ϕ = angle of repose.

Those familiar with trigonometry will easily be able to deduce this short cut, and the deduction will serve the useful purpose of impressing it upon the memory.

The horizontal thrust is then turned into an equivalent thrust parallel to the angle of repose, either graphically or trigonometrically, by multiplying by the secant of the angle of repose. The attacking forces being now calculated, it is possible to determine the necessary strength of the defending forces, *i.e.*, the section of the wall. This section must be arrived at by trial and error—a section being assumed and calculated to see whether it is strong enough or too strong. The assumed section of the wall is drawn, its centre of gravity is ascertained (see Chapter V.), and a vertical line is drawn through it. The two forces of the inclined thrust and the weight of the wall can be now combined by a parallelogram of forces, and the point where their resultant will cut the base can be fixed. Then it must be ascertained whether the angle which the resultant makes is within 20 per cent. of the limiting angle of friction, and if, as generally happens, it is found that there is no danger of sliding anywhere; the next thing is to see how near the resultant is to the toe. If it is outside the middle third a stronger section would generally be tried, but if the circumstances justify a reliance on the power of the joints to resist tension, it must be ascertained that the negative pressure is not greater than the limit of tenacity which they can be relied upon to exert. Then the maximum pressure at the toe is ascertained, and if that is well within the crushing strength of the brickwork or masonry, the design of the footings and foundations can be proceeded with, having regard to the supporting power of the soil and the transverse strength of the projecting concrete (see Chapter XX.).

NECESSITY FOR DRAINING BACK OF RETAINING WALLS.—An important point which must not be forgotten is the necessity of well draining the back of the wall, for water there will not only rot the wall but will exert a fluid pressure of its own and also cause the back of the wall to act as if it were smooth. For this purpose a backing of stones or brick rubbish is generally filled and rammed behind the wall, and 3-in. agricultural drain pipes are built into the wall to "weep" out any collection of percolating moisture to the face. A usual allowance is one weep hole per 2 yds. super. of face, or, say, five or six holes per square of 100 ft. super.

TYPICAL SECTIONS OF GRAVITY RETAINING WALLS.—Fig. 124 shows the proportions of typical sections of gravity retaining walls commonly adopted in practice.

According to Sir B. Baker the thickness of an earth retaining wall for average earth backing and foundations need not exceed one-third the height of the wall measured from the footings, and a wall one-quarter of its height in thickness, with a batter on one or two inches per foot on the face, possesses sufficient stability when the backing and foundations are favourable; and further, that under no conditions of surcharge or heavy backing is it necessary to make a retaining wall with solid foundations more than one-half of the height in thickness.

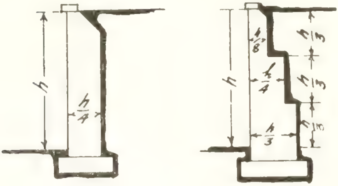


FIG. 124.

WEIGHT OF BUILDINGS NEAR THE BACK OF RETAINING WALLS.—When the foundations of any buildings are carried by the wedge of earth supported by a retaining wall, the weight they carry must be added to the weight of the wedge in estimating the horizontal thrust.

LIMITS TO BATTER OF RETAINING WALLS.—It will be seen that if the bed joints and foundations of a retaining wall are inclined, the danger of sliding is considerably reduced. Owing, however, to the tendency of wet to lodge in and destroy the joints of walls built battering, retaining walls are seldom built to a greater batter than 1 horizontal to 6 vertical.

CHAPTER XX

FOUNDATIONS

Safe Pressure on Foundations—Impossibility of Preventing Settlement—Stresses in Projecting Footings—Omission of Concrete on Hard Soils.

THE foundations of retaining walls are designed upon the same principles as other foundations, and may be considered simultaneously.

SAFE PRESSURE ON FOUNDATIONS.—There is no subject upon which so much honest difference of opinion exists, and there is no subject upon which it is more dangerous to dogmatise. Lists of safe loads upon various soils are given by various authorities, and are to be found in the standard rules adopted by railway companies, Government departments, etc.

The following is an average of different authorities at ordinary depths:—

	Tons per Square Foot.
Earth, ordinary firm	1
Ordinary clay	2
Hard compact clay	4·5
Ordinary gravel	3
Compact gravel	4·8
Hard chalk	6
Rock	16
Ordinary brickwork in cement mortar	5
Hard brick (including London stocks) in cement mortar	8
Blue brick in cement mortar	12
Lime concrete (good)	8
Cement concrete (4-2-1)	12

The difficulty in the way of adopting any set of fixed safe loads for general practice is that almost every case differs. Railway companies, who usually deal with deep foundations, can standardise heavier loads than would be safe at shallower depths. The depth of a foundation bottom is a most important point—the deeper the bottom the less chance there is of displacing vertically and laterally the soil below it. The usual assumption is that the bearing power of any soil increases directly with the depth. Thus if a soil is capable of taking $\frac{1}{6}$ ton per square foot at the surface it could bear 1 ton at 6 ft., 2 tons at 12 ft., and so on. But this means depth in that particular soil—it would not apply to foundations in, say, gravel overlaid by 6 ft. of loam; and it is a safe rule to calculate as depth only such depth as consists of good solid earth.

Having decided upon the pressure to which the soil can be subjected, the next question is how to so distribute the weights to be carried that this pressure may not be exceeded.

Impossibility of Preventing Settlement.—It must be remembered that all foundations, except rock, allow settlement, and that the object to be aimed at is not the *prevention* of settlement, which is an impossibility, but to secure an even settlement at every point. In soils of equal stability all over the site, this is obtained by proportioning the areas of the foundations so that, as nearly as possible, the same pressure per square foot is placed on the soil. If the soil is irregular with pockets of soft material here and there, the problem becomes difficult and must be treated according to circumstances—the usual object being to bridge over any soft pockets. Riverside foundations, for instance, often present very difficult features, such as a general bottom of hard gravel with pockets of sand and of soft peat occurring close together. No rules can be given for such cases, but a general safe guiding principle is to disturb the soil as little as possible and to prevent any lateral movement. Frequently on a riverside it is cheapest to drive sheet piling all round the site and build on a concrete raft. This can usually be done more cheaply with reinforced concrete piles and a reinforced concrete raft than with the old method of timber piles on a concrete raft 4 or 5 ft. thick. One method of bridging over a soft place occurring in the run of a trench is to place in the bottom of the concrete angle or tee irons or rolled joists bridging the soft places, and thus converting the length of concrete over the soft place into a massive reinforced-concrete beam.

It should always be remembered that cracks occur through *unequal* settlement, and are just as likely to be caused by one part of the foundation being made too strong and stiff as by a part being too weak.

STRESSES IN PROJECTING FOOTINGS.—The whole thing generally resolves itself into a question of how much projection shall be given to the concrete footings. There is a certain weight from floors, roof, brickwork, etc., coming on the base of the wall, and the base usually (but not always) has to be increased by concrete footings so as to spread the load over a greater area of soil. Then the question immediately arises as to what is the safe relation between depth of concrete and projection. The problem is a simple one if, say, a foot run of wall be considered, and the concrete projection be regarded as a cantilever carrying an equally distributed load consisting of the reaction of the earth ; the length of the cantilever being the projection, its breadth 12 in. (1 ft. run of wall), and its depth the thickness of the concrete (Fig. 125). It will be seen at once that the greater the pressure called up from the foundations the greater will be the depth required for any given projection. Concrete is weak and unreliable in tension and the flange stress that can be allowed for in a good cement concrete cantilever should not exceed 50 lbs. per square inch. Taking the case of a pressure of, say, 1 ton per square foot on the soil, and cement concrete, let p = projection, d = depth.

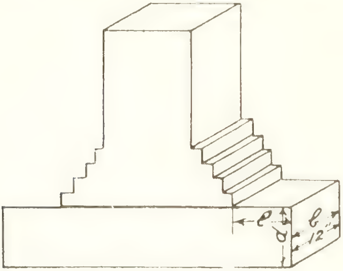


FIG. 125.

1 ton per square foot $= \frac{2240}{12}$ 186 lbs. on a strip 12 in. long × 1 in. wide, *i.e.*, the load which will come on every 1-in. run of cantilever.

$$\text{B.M} = \text{M.R.}$$

$$186 \times p \times \frac{p}{2} = \frac{fbd^2}{6} = p^2 \frac{186}{2} = \frac{50 \times 12 \times d^2}{6}.$$

$$p^2 = \frac{d^2 \times 50 \times 12 \times 2}{6 \times 186} = \frac{200d^2}{186}.$$

Approximately $p^2 = d^2$ or $p = d$.

So that when using good cement concrete, and stressing the ground up to 1 ton per square foot, the concrete would be strong enough if the depth be made equal to the projection, 6 in. deep for 6 in. projection, 9 in. deep for 9 in. projection, and so on.

When using good lias lime concrete capable of exerting a safe *f* of, say, 35 lbs. per square inch, the projection should be kept within two-thirds of the depth, *i.e.*, the projection having been fixed, the depth is made one and a half times the projection, Fig. 126. When any considerable spread of concrete is required it will generally be found less expensive to reduce the tensile flange stress by reinforcing the bottom edge of the concrete than to do so by increasing the depth.

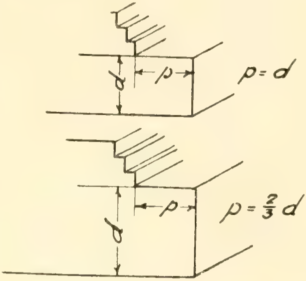


Fig. 126.

Omission of Concrete on Hard Soils.—When building on hard chalk or exceptionally hard gravel, concrete foundations are often omitted, because the ground can carry more than the greatest load which can be transmitted to it by ordinary brick footings.

Thus if the load on walls is kept under 8 tons per square foot, the pressures on the underside of the base of the footings (which double the width of the wall) cannot exceed 4 tons. There is therefore no need to spread this any further by means of concrete over a base capable of taking safely a load of 6 tons per foot super. or over.

PART IV

ROOFS

CHAPTER XXI

ROOFS

GRAPHIC SOLUTION OF STRESSES

Flat Roofs—Inclined Roofs—No Thrust in Lean-to Roofs—Stresses in Inclined Beams or Rafters—Stresses in Simple Roof Couple—Measurement of Stresses in Model Roof Couple—Stresses in Collar Roof—Stresses in Braced Roof Truss—Incorrect Assumption of Frictionless Joints, Underlying all Graphic Solutions.

FLAT ROOFS.—The simplest form of roof, flat bearers supporting a suitable roof covering, is subject only to the same stresses which are met with in beams, but any inclination of the roof from the horizontal in order to throw off the water introduces fresh stresses, which in large roofs are apt to become somewhat complicated.

INCLINED ROOFS.—The flat roof in Fig. 127 (*a*) is obviously subject to beam stresses only, but in a sloping roof over the same span, Fig. 127 (*b*), a surface is presented to any wind blowing from A to B, which tends to overturn the walls, and that tendency must be resisted.

NO THRUST IN LEAN-TO ROOFS.—At first sight there seems to be also a tendency for the weight of

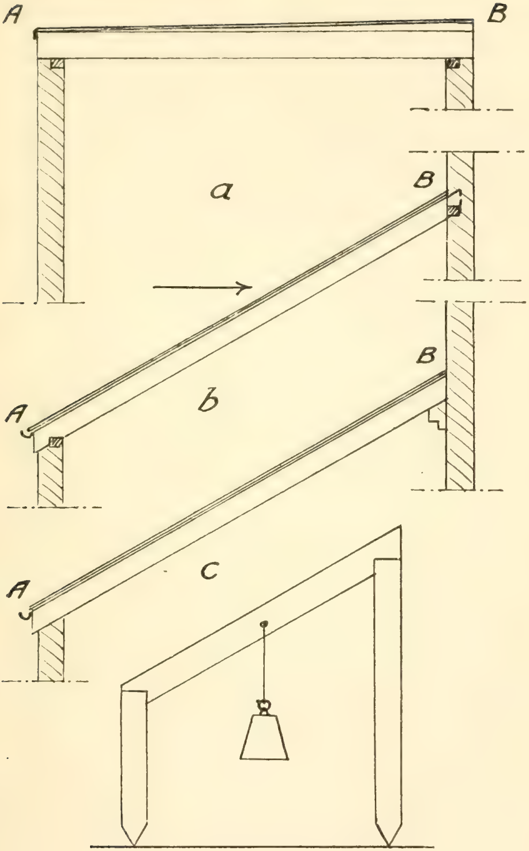


FIG. 127.

the roof to push out the walls at A. This would be the case if the roof were constructed as in Fig. 127 (*c*), in which case the tendency must be resisted by firmly securing the rafter ends at B to a plate let into the wall or resting vertically on an offset. But so long as the ends of rafters or inclined beams or girders are provided with horizontal seatings they do *not* tend to push out laterally. This can easily be proved with pieces of notched wood on square-topped supports resting on pointed ends. If the bearings are not horizontal, Fig. 128, a condition of things which is most unlikely in practice, the tendency to sliding or lateral push can be determined as follows:—An inclined bearing would resist the vertical force partly by means of a reaction at right angles to its surface, and partly by friction in a direction parallel to its surface. A vertical line ab, Fig. 128, of a length corresponding to the vertical reaction, and a line ac at right angles to the slope, both drawn from the centre of the bearing, would form two sides of a triangle of forces which would be completed by a line bc parallel to the slope. The length of bc would represent the push on the wall which would have to be resisted. If the complement δ of the angle of slope ϕ were less than the limiting angle of friction between the materials of the bearing, the beam would, of course, simply tend to slide downwards, and bc would be the measure of that tendency.

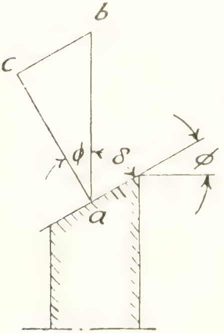


FIG. 128.

STRESSES IN INCLINED BEAMS OR RAFTERS.—Inclined beams or rafters are in the condition of beams under transverse stress as well as being under a certain amount of direct stress, viz., tension from the upper bearing to the centre of the beam, and compression from the centre to the lower bearing. The amount of this direct stress depends upon the angle of the slope and can be determined graphically.

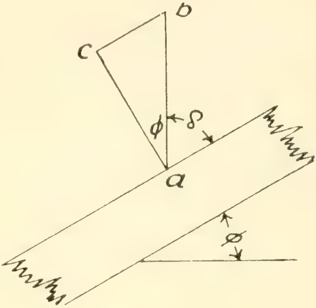


Fig. 129.

If *b a*, Fig. 129, equals the vertical shear at any given section of an inclined beam, then *a c* represents the shear at right angles to the beam, and *c b* represents the direct stress on the section.

This would be expressed trigonometrically as Direct stress $= ba \sin \phi$, where $\phi =$ the angle which the slope of the beam makes with the horizontal. The section is also subject to a bending moment, which is calculated first as for a beam, viz., the reaction from either bearing × its *horizontal* lever arm minus the contrary moments due to the various elements of the load between the section and that bearing multiplied by their respective *horizontal* lever arms. The lever arms are measured horizontally because the loads and reactions generating these bending moments are acting vertically, and their lever arms must (see Chapter V.) be measured at right angles to their direction. Then the extra fibre stress f at the section in question can be determined by the usual formula for eccentric loading (page 223):—

$$\begin{matrix}\text{Max. } f, \\ \text{Min. } f,\end{matrix} = \frac{W}{A} \pm \frac{M}{Z}.$$

In which W = the direct compression or tension at the section,
M = the bending moment at the section,
A = the area of the section,
Z = the section modulus.

It will be noted that as the slope approaches the vertical, the direct stress increases and the bending moment—as it decreases with the decreasing span—becomes less important. On the other hand, when the slope is flattened and the conditions approach that of a horizontal beam, the direct compression becomes very small, and nearly the whole of the fibre stress is due to the bending moment.

Stresses in Simple Roof Couple.—In the case of the couple roof, shown in Fig. 130, there is no

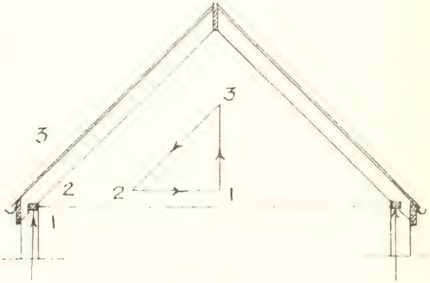


Fig. 130.

horizontal bearing at the ridge, and the rafters are practically free to turn about that point as on a hinge. Each rafter foot will, therefore, tend to thrust outwards. The extent of this thrust can be found by a triangle of forces, as explained on page 80—1-3 being the upward reaction, 2-3 the thrust in the rafters, and 1-2 the outward thrust. If *each* rafter thrust outwards with a force equal to $1\frac{1}{2}$, the tension in the horizontal tie which would neutralise these two thrusts would be equal to *one* of them, not both, viz., $1\frac{1}{2}$ not 3.

MEASUREMENT OF STRESSES IN MODEL ROOF COUPLE.—This can be tested in a model roof couple with compression and tension balances introduced into the rafters and tie, Fig. 131; it should be remembered that the slope of the rafters in such a model varies as the balances are lengthened or shortened under the load, and the angle of slope must, therefore, be measured after the model is loaded. It makes no difference to the reaction whether the load be concentrated at the apex or distributed in any way along the rafter so long as it is arranged symmetrically about the apex; and as the thrust in the rafter and the tension in the tie are caused by the reactions and not directly by the

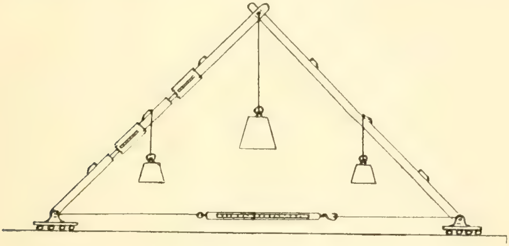


FIG. 131.

load, they also remain the same, in an unbraced roof, whether the load be applied at the ridge, along the rafters, or close to the shoes, so long as it is symmetrical about the ridge. The truth of these important points should be tested by actual experiment. Internal bracing would, of course, affect the stresses in the braced portions of the truss, but not the stresses at the rafter feet.

STRESSES IN COLLAR ROOF.—The roof shown in Fig. 132, although unbraced below the collar, acts

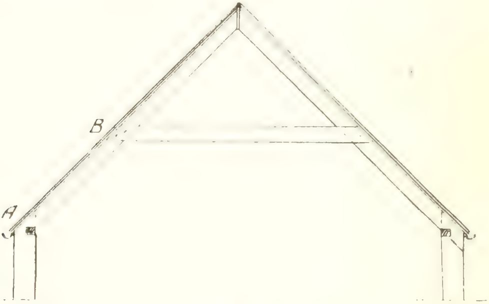
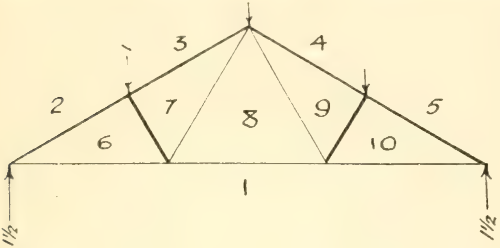


FIG. 132.

practically as a completely braced truss, for until the rafter has been broken across below the collar the roof can only exert on the walls the very restricted thrust which is permitted by its own deflection. As soon as the cantilever A B has finished deflecting, the roof will act in the same way as if the rafter feet had been secured with a horizontal tie.

STRESSES IN BRACED ROOF TRUSS.—The stresses in more complicated trusses can generally be determined graphically by a simple extension of the principle of the triangle of forces. Let the roof truss shown in Fig. 133 be loaded at the ridge and purlin points with loads represented by 1, which loads include the weight of the truss. The reactions at each bearing will obviously be $1\frac{1}{2}$. All the spaces between the external loads and the paths of the internal stresses (which stresses must, of course, travel along the struts and ties of the frame) are first numbered. The loads are then set down vertically to any convenient scale, the reactions are found if necessary by funicular polygon (Chapter VIII.), or in cases of regular loading are simply measured back to give point 1.



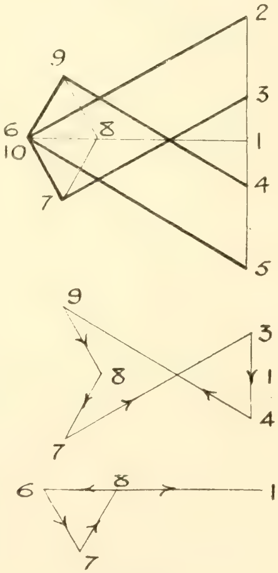
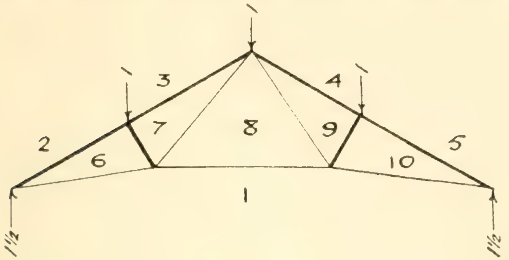


FIG. 133.

Any convenient triangle of forces can then be drawn to start the diagram, and from its angles fresh diagrams are built up until the whole stress diagram is complete. For instance, a line from 1 parallel to 1-6 and another from 2 parallel with 2-6 intersect at and give the point 6. From 6 a line can then be drawn parallel to 6-7, which will intersect at the point 7 on the line parallel to 3-7. Then from the point 7 a line can be drawn parallel to 7-8 to intersect on the horizontal line from 1, and so on until the figure is complete. The lengths of the various lines measured to the same scale as the line of loads represent the amounts of the forces acting in each member. If the lines in the stress diagram which denote compression be thickened, they will be found to follow round the diagram in a systematic manner which generally enables the tension and compression members to be easily recognised. If, however, there be any doubt as to a particular member, then from the general diagram there can be selected those lines which go to form the diagram of the forces which are acting at *one end* of the member in question. As explained in Chapters VII. and VIII., *one* arrow head on such an extracted force diagram gives the direction of all the other forces. Take, for instance, the bar 7-8. Of the forces acting at the upper end, the load 3-4 is obviously acting downwards on the point. If the diagram of the forces acting at the point be extracted from the main stress diagram, an arrow head on 3-4 pointing downwards starts a train of similar arrow heads following round the diagram; and this at once establishes the fact that 7-8 and 8-9 are pulling away from the point and therefore are in tension. Similarly, the bars 3-7 and 4-9 are pushing on the point and are in compression. The forces at the lower end of 7-8 *appear* to form an unclosed diagram, but this is merely because the force 6-1 is lying along 8-1, and is for that reason not distinguishable from it. This is more clearly shown in Fig. 134, which shows a similar diagram for a roof with a cambered tie-rod. In this the direction of 6-1 not being horizontal like 8-1 can be easily distinguished from it on the stress diagram.



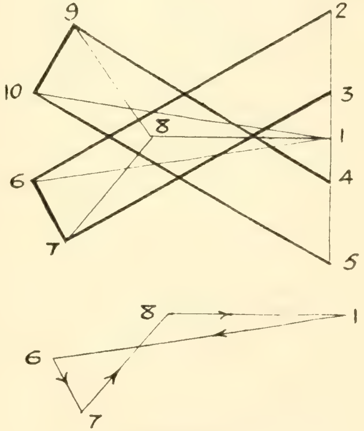


Fig. 134.

MEASUREMENT OF STRESSES IN MODEL BRACED ROOF TRUSS.—The graphic method of determining the stresses in roof trusses is so simple and speedy that it is advisable for the student to prove for himself both its accuracy and its limitations on a scale model on which the loading can be varied, and in which all the stresses set up by any system of loading can be measured. Such a model is shown in Fig. 135. It is built up of light laths forming the compression members, and tension members of cords carrying light tension spring balances and adjustable as to length. The joints are small loose screw bolts. When such a model truss is loaded the distortion caused by the action of the spring balances is considerable. It is therefore hung in front of a skeleton diagram, and after loading, the various cords are tightened up until the different members are pulled back into their correct position. The tensional stresses

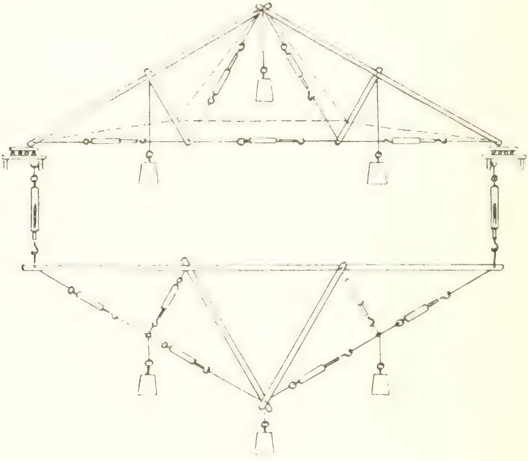


FIG. 135.

can then be read off on the balances. The practical difficulty of readily taking up the distortion of compression balances is overcome by using a similar model inverted as shown. By this means all the stresses are reversed in direction, but remain unaltered in amount, and the compression members become ties and can be measured in the same way as before. A very high degree of accuracy is possible with such a scale model, costing, exclusive of balances, only a few pence.

The balances are the ordinary Salter " Sportsman's " type, reading up to 8 lbs. by 2 oz., and cost about two shillings each. One of the most useful properties of such a model is that it at once shows the fallacy which underlies the assumption—made in all calculations of framed structures—that the joints are frictionless hinges. If the small screw bolts be tightened to even a moderate extent, the stresses in some of the members are considerably altered, which is also the case when the bearings are not made practically frictionless by the rollers shown. In practice, joints, especially riveted joints, are never in this state. Although the result of incorrectly assuming them to be so is generally on the safe side, it is necessary to recognise the fallacy and to bear it in mind.

CHAPTER XXII

ROOFS (*continued*)

METHOD OF SECTIONS

Liability of Graphic Methods to Break Down—Calculation by Moments of Leverage—Method of Sections.

LIABILITY OF GRAPHIC METHODS TO BREAK DOWN.—It occasionally happens that the graphic method of calculating roof stresses, being based upon the continuous building up of triangles of forces, breaks down at some point where more than one force is unknown. Take, for instance, the roof truss shown in Fig. 136. After proceeding to a certain extent with the diagram as shown, one is brought to a sudden stop by the impossibility of locating the point 16, without which the remaining triangles cannot be drawn.

METHOD OF SECTIONS.—It is then necessary to go back to the simple principles of leverage upon which graphic statics are based, and of which they are after all merely a convenient abbreviation, and to calculate the stress in bar 9-16 through the medium of the turning moments which generated that stress. Although the calculation of entire roof trusses by calculating moments with the aid of what is known as the method of sections would be unnecessarily tedious, yet on the other hand calculation by moments is by no means to be despised, for it is not only invaluable when the speedier methods of graphic statics break down, but also it forms the most speedy method of arriving at the stresses in any one member, and therefore affords a ready and convenient check upon the accuracy of any large and complicated diagram. The general principles of the method are

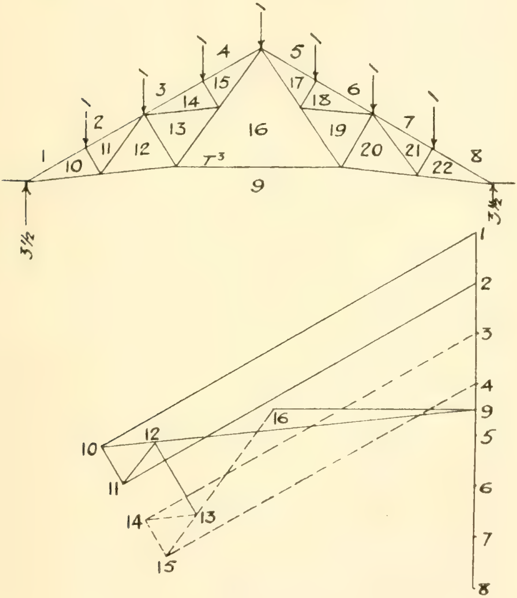


FIG. 136.

very simple. It is first assumed that the framed structure is cut completely in two on a line of section which is not necessarily a straight line, but which passes through if possible two other bars in addition to the one which it is required to investigate. In Fig. 137 part of the roof truss under consideration is shown thus detached and isolated for examination, together with the loads and the main reactions and the stresses which previously acted upon it. It is, therefore, in precisely the same state of equilibrium and of internal stress as it was before severance. The loads, stresses, and reactions in the remainder of the structure do not concern it, for whatever they were, whether great or small, simple or complicated, their total effects were all compounded and paid for in a lump sum in the form of *the main reaction*. It will be obvious that the portion thus isolated could only be kept in position and in the same state of equilibrium that it was before severance—*if forces were applied to the ends of the severed bars equal to the stresses which existed in those bars when in their unsevered state.*

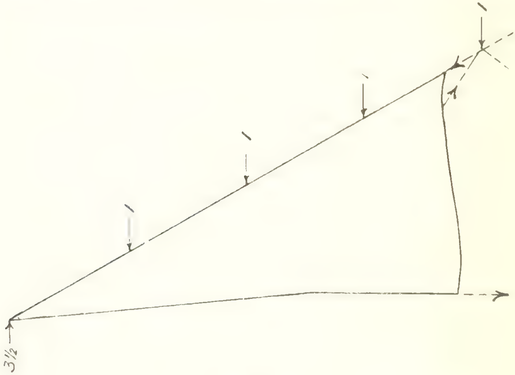


FIG. 137.

It will be found somewhat more convenient to consider it now not as a framework, but as being (which for all practical purposes it is) a flat rigid plate acted upon by several forces, some of them of known and some of them of unknown extent, but all acting in known directions. In fact, as being somewhat in the same condition as the piece of cardboard shown in Fig. 15, page 47. As there are several unknown forces, triangles or polygons of forces—which will only determine the amount of two unknown forces—cannot be used. But under the action of the forces acting, viz., the main reaction, the loads and the original stresses in the severed bars, the plate is in equilibrium, and the turning moments of those forces, therefore, balance each other about every possible point upon which it could rotate. A force acting upon a plate and tending to rotate it exercises no turning moment about any point of rotation along the path of its own direction, for about such a point it would have no leverage to work with. As the forces balance about *all* points they must also balance about a point which is situated in the line of direction of one of their number, and they must also so balance without any help from that force. They must similarly balance about a point situated on the lines of direction of two of their number, and must do so independently of these two forces ; therefore about a point situated on the intersection of the direction of two of the unknown forces, balance is independently maintained by the remainder of the forces, and of these the direction of all and the extent of all but one are known. The moments forming the balance can, therefore, be expressed in the form of a simple equation containing only one unknown factor. Lines having been drawn at right angles to the forces, and passing through the assumed point of rotation, the lever arms can be scaled off and the clockwise and anti-clockwise moments set down as an equation, of which the solution provides the missing force.

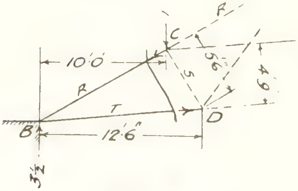


Fig. 138.

If it were required to find the stress in the rafter R, Fig. 138, between the purlin and the shoe. In this case a section through two bars will suffice. Moments would be taken about the point D, through which the unknown force in the tie D acts. The balanced turning moments about D are:—

↻ Clockwise. = Anti-clockwise. ↺

Reaction at B $(3\frac{1}{2}) \times 12$ ft. 6 in. $= \text{R} \times 5$ ft. 6 in.

$$\therefore \quad \text{R} = \frac{3{\cdot}5 \times 12{\cdot}5}{5{\cdot}5} = 8.$$

If it had been required to determine the force in the tie T, moments would be taken about the point C, so as to keep the unknown force in R out of the equation, which would read—

↻ Clockwise. = Anti-clockwise. ↺

$3\frac{1}{2} \times 10$ ft. = T × 4 ft. 9 in.

$$\therefore \quad \text{T} = \frac{3{\cdot}5 \times 10}{4{\cdot}75} = 7{\cdot}35.$$

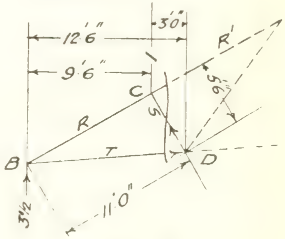


Fig. 139.

To find the stress in the strut S, a section would be taken as in Fig. 139, and moments taken about the point B about which the forces in R, T, and the reaction have no turning moment. The equation would therefore read—

↻ Clockwise. = Anti-clockwise. ↺

$$\text{Load at C }(1)\times 9\text{ ft. }6\text{ in.} = S\times 11.0$$

$$\therefore\ S = \frac{9{\cdot}5}{11.0} = 0.865$$

To find the stress R′, the point D, which disposes of T and S, would be selected, and the balance would be represented by—

↻ Clockwise. = Anti-clockwise. ↺

$$\text{Reaction B }(3\tfrac{1}{2})\times 12\text{ ft. }6\text{ in.} = \text{Load at C }(1)\times 3\text{ ft.} + R'\times 5\text{ ft. }6\text{ in.}$$

$$\therefore\ R' = \frac{(3{\cdot}5\times 12.5)-3}{5{\cdot}5} = \frac{43{\cdot}7-3}{5{\cdot}5} = 8.15.$$

Care should be taken in placing the arrow heads showing the direction of the forces, which are equivalent to the stresses in the severed bars. If the original stress were compression, as in a rafter or strut, the arrow point should point towards the severed end. If it be tension, as in a tie, it should indicate a force pulling away from it.

Now let the method be applied to the solution of the difficulty over which the graphic method broke down, viz., the tensions in the tie T^3, Fig. 136. A convenient section, Fig. 140, would be taken through the three bars shown, two of which pass through the apex A, which would be selected as the point round which the moments would be calculated, as follows :—

↻ Clockwise. = Anti-clockwise. ↺

$$\text{Reaction at B }(3\tfrac{1}{2})\times 37\text{ ft.} = T^3\times 19\text{ ft.} + (1\times 9\text{ ft.}) + (1\times 18\text{ ft. }6\text{ in.}) + (1\times 28\text{ ft.}).$$

$$= 3\tfrac{1}{2}\times 37 = (T^3\times 19) + 9.0 + 18.5 + 28.0.$$

$$\therefore\ T^3 = \frac{(3{\cdot}5\times 37)-55{\cdot}5}{19} = \frac{130-55{\cdot}5}{19} = 4.$$

If a length equal to 4 be now scaled off on the

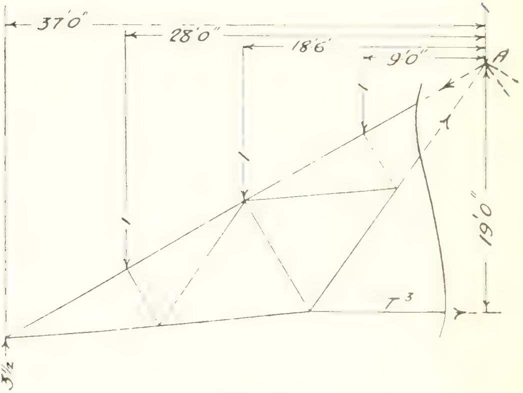


FIG. 140.

horizontal line from 9 in the stress diagram, Fig. 136, the extremity of it will be point 16, from which a line can be drawn parallel to 13-16, intersecting the line from 12 on point 13, and the diagram could be completed.

The student is advised not to rest contented with mentally following this explanation, but to draw out a good large truss, such, for instance, as the one shown in Fig. 141, to a different scale and with different

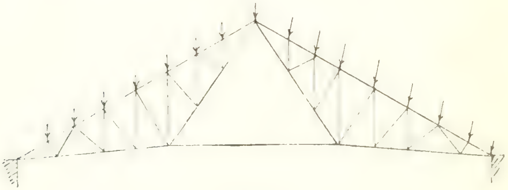


FIG. 141.

loads, and to determine the stress in every member both graphically and by the method of sections; and then, if possible, construct it with laths and string and test each stress with a light spring balance, as described in the last chapter.

The method of sections is, of course, applicable to any framed structure.

If it be desired to ascertain the stress in the upper boom B of the lattice girder in Fig. 142, moments would be taken about C, the equation reading—

$$\circlearrowright\ \text{Clockwise.} = \text{Anti-clockwise.}\ \circlearrowleft$$

$$10 \times 16 \quad = \quad (B \times 8) + (2 \times 8).$$

$$\therefore\ B = \frac{(10 \times 16) - (2 \times 8)}{8} = 18.$$

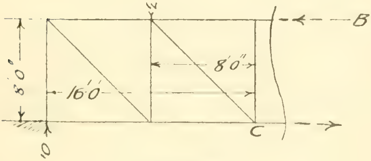


FIG. 142.

CHAPTER XXIII

WIND PRESSURE

Difference of Pressure on Small and Large Surfaces—Factor of Safety for Wind Pressure—Wind Pressure Measured as a Live Load, but not Doubled—Observed Effect of High Winds—Horizontal Pressure for Assumption in Ordinary Roofs—Effect of a Force Applied Obliquely to a Surface—Effect of Negative Drag on the Leeward Side of a Roof—Hutton's Formula—Duchemin's Formula—National Physical Laboratory Tests—Effect of Height upon Wind Pressure.

DIFFERENCE OF PRESSURE ON SMALL AND ON LARGE SURFACES.—The subject of wind pressure has received much attention from scientists, but unfortunately most of the results arrived at have been deduced from the maxima records of small isolated anemometers, and more recent experiments upon large boards tend to show that the maximum pressure upon a small surface is likely to be very considerably higher than the highest average pressure over a larger surface, whether the smaller surface forms a part of the larger or not.

This probably explains why buildings designed to withstand wind pressures of 40 or 50 lbs. per foot super. (a very common assumption in ordinary practice) are frequently so much more massive, expensive, and ugly than mediæval buildings of the same class, which were designed before anemometers were invented, and upon the basis of experience only, but which have withstood the storms of centuries without causing their custodians any anxiety.

The author would be the last to advocate the relaxation of rules which have been found to be safe—but the assumption of wind pressures of over 30 lbs. horizontal in England over large roof areas has no reliable scientific basis; it is contrary to the results of years of record in a most exposed situation—the Forth Bridge—and it cannot be reconciled with ordinary experience.

FACTOR OF SAFETY FOR WIND PRESSURE.—If a factor of safety of 4 be used, it can hardly be necessary to provide for any greater wind pressure than one which would produce effects of a distinctly unusual character. This would leave a margin of 100 per cent. between an exceptional wind, acting at very brief and infrequent intervals, and such a wind pressure as would cause the elastic limit to be reached in the materials of a structure resisting it.

WIND PRESSURE MEASURED AS A LIVE LOAD, BUT NOT DOUBLED.—It is sometimes suggested that wind pressure, being a live load, should be doubled in order to obtain the true effect of a live load. But every assumed wind pressure can only be deduced from the *effects* of wind on materials, and the suggestion that it must be doubled in order to arrive at itself is merely ridiculous.

OBSERVED EFFECT OF HIGH WINDS.—The effects of unusually high winds in this country may be studied in the overturning of railway carriages and the destruction of walls and chimneys, whilst the effects of wind pressure on pedestrians and upon doors are also instructive.

The overturning of railway carriages can scarcely be considered as a frequent occurrence, and the moment of stability of carriages which have been overturned has been found to have been sufficiently high to have necessitated a pressure equivalent to about 30 lbs. per foot super. on the windward side of the carriage, if any negative pressure or suction on the leeward side be neglected, but this negative pressure would probably form one-third to one-half the total overturning moment. The fact that lighter carriages in the same train have kept to the rails is additional proof of the local character of unusual gusts.

In every town, almost in every street, can be found examples of old walls and chimneys which must inevitably have blown over had they been at any period of their existence subjected to pressures far less than this. Take the case of a 9-in. garden wall, 7 ft. high, built with lime mortar. The tenacity of such mortar is practically *nil*, the wall standing only by virtue of the moment of stability due to the weight of its parts. This would be $7 \times 0.75 \times 0.375 \times 112$ lbs. per foot run of wall, and the overturning moment of a wind pressure of x lbs. per foot super. would be $x \times 7 \times 3.5$ ft.-lbs. per foot run of wall.

The wall would, therefore, be blown down at any pressure exceeding

$$\frac{7 \times 0.75 \times 0.375 \times 112}{7 \times 3.5} = 9 \text{ lbs. per foot super.}$$

A brick chimney, one flue (18 in.) thick, weighs with the flues deducted, about $\frac{3}{4}$ cwt. per foot cube, and would presumably be safe if built according to the rules of the London Building Act, viz., to a height of six times its base, or 9 ft. The joints of such a chimney would be even weaker than those of a 9-in. garden wall. Its moment of stability would be $84 \times 1.5 \times 1.0 \times 9 \times 0.75$ ft.-lb. per foot run, and the pressure per foot super. on one face which would overturn it would be—

$$\frac{84 \times 1.5 \times 1.0 \times 9 \times 0.75}{9 \times 1.0 \times 4.5} = 21 \text{ lbs. per foot super.}$$

It is a well-known fact that the negative pressure or suction on the lee side of a thin flat surface exposed to wind may equal or even exceed the direct pressure on the windward side, so that direct pressures of 5 to 10 lbs. might well suffice to overturn such walls and chimneys.

A wind which would blow over a strong man weighing, say, 150 lbs., and leaning forward with his legs braced so as to gain an effective base of, say, 1 ft. 6 in., might fairly be regarded as exceptional. The body, head, and legs of such a man would present a surface of about 8 sq. ft. to the wind, and the centre of gravity of that projected surface would be about 3.6 ft. above the feet. The effect of any negative suction at the back would be about equal to the relief afforded by the somewhat curved nature of the projected surface. The greatest wind pressure which such a man could stand up to would, therefore, be $\frac{150 \times 1.5}{8 \times 3.6}$, or 7.8 lbs. per foot super.

Another fair example of an exceptional gale would be one against which a powerful man could not open an ordinary door when using all his force. The limit of horizontal push which a powerful man can exert at chest level has been found to be about 70 lbs.

An ordinary door would present a surface of 20 ft. super., and half of any pressure on this surface would, on the door being pushed open, be borne by the hinges. There would in this case be no negative pressure to allow for, and the wind pressure which would resist the utmost efforts of a strong man would be $\frac{70 \times 2}{20}$, or 7 lbs. per foot super.

NATIONAL PHYSICAL LABORATORY MEASUREMENTS OF WIND PRESSURE.—These homely deductions from common experience, and others of a similar nature, prepare one for some of the results which have been obtained by careful and exhaustive experiment at the National Physical Laboratory upon the relation between wind velocity and pressure. (See *Proc. Inst. C.E.*, vol. clxxi.)

These may be summarised as follows :—

Velocity in Miles per Hour.	Pressure per Square Foot in Lbs.	Remarks.
10	0.32	Gentle.
20	1.28	Light breeze.
30	2.88	Moderate wind.
40	5.12	High wind.
50	8.00	Gale.
60	11.52	Storm.
70	15.68	Heavy storm.
80	20.48	Violent storm.
90	25.92	Hurricane.
100	32.00	Violent hurricane.
120	46.08	...
140	62.72	...

HORIZONTAL PRESSURES FOR ASSUMPTION ON ORDINARY ROOFS.—When a wind pressure of only 8 lbs. per foot super. is regarded as that produced by a gale of 50 miles an hour—one which would overturn fairly weak walls and chimneys, blow down chimney pots wholesale, blow strong heavy men over, and prevent a strong man from opening an ordinary door—then our preconceived ideas of wind pressures are apt to appear a little out of proportion, to say the least. It does not, therefore, require much scientific investigation or mathematical deduction to understand why buttresses designed to meet uniform wind pressures of, say, 50 lbs. per foot look decidedly clumsy when compared with those evolved by the experience of the mediæval builder.

Moreover, natural wind pressure does not exert its maximum force over all portions of a large roof at the same time; and the author suggests that at least for buildings in ordinary situations,* a rule deduced from the Forth Bridge records is sufficient, viz., to assume a steady wind pressure of 30 lbs. per square foot on any area of 300 sq. ft. and under, and to reduce this by 1 lb. per square foot for every 100 sq. ft. in excess of 300 to a minimum of 20 lbs. per square foot.

The principle of a 30-lb. wind is accepted in the L.C.C. Gen. Powers Act, 1909, Part IV., and is in accordance with the building laws of large Continental and American cities.

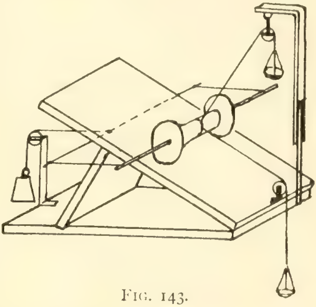


Fig. 143.

Effect of a Force Applied Obliquely to a Surface.—When a force is applied to a surface in such a manner that there is no friction between the surface and the medium through which the force is applied, then the surface can only offer reaction in a direction normal, viz., (at right angles) to itself. This can easily be proved by threading a couple of similar empty cotton reels on a knitting needle, and applying pressure to a piece of smooth inclined board by means of silk loops, as shown in Fig. 143. In whatever direction the force be applied, the trolley will tend

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* See also page 270.

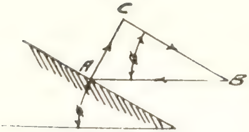


Fig. 144.

to run up or down the slope until the silk loops are at right angles to the slope. If the tendency towards upward or downward movement be measured, it will be found that a force A B, Fig. 144, in a direction other than normal to the slope will be balanced by two components, one, C A, normal, and one, B C, parallel to the slope.

Effect of Negative Drag on the Leeward Side of a Roof.—It is sometimes suggested that the action of a horizontal wind pressure on a roof slope results in two such components, one of which represents a deflected pressure which escapes harmlessly up the roof slope, the only demand made upon the stability of the roof being represented by the normal component C A, and equal to A B sin ϕ. But unfortunately for this simple solution, roof slopes are not smooth, and the component B C also is not harmless. It represents a stream force which tends to curl over the ridge, and to exert a negative pressure or suction on the lee side, Fig. 145.

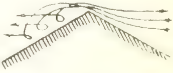


Fig. 145.

The negative pull on the leeward side is well established by a large number of experiments made at the National Physical Laboratory at Teddington to determine the relation between horizontal wind pressure and the pressure per foot super. on inclined roof surfaces. Considerable information as to these experiments is recorded in two lengthy papers by Dr Stanton in the *Laboratory Journal.*

It has not been customary hitherto to make any allowance in roof design for negative drag on the leeward side, and the efforts of authorities have been devoted to establishing the correct relation between a given horizontal wind pressure P and the pressure P^N per square foot of sloping surface which P induces in a direction normal to the slope.

The simple solution shown in Fig. 144 is very seldom used, and the two best known formulæ are Hutton's and Duchemin's.

HUTTON'S FORMULA.—Hutton's formula is as follows:—

$$P^N = P\sin\phi^{1.84\cos\phi - 1}.$$

Assuming a horizontal wind pressure P of 30 lbs. per foot super., it gives the following results for three typical angles of roof slope:—

Angle of Roof Surface.	30°.	45°.	60°.
Windward pressure, P^N -	20 lbs.	25.5 lbs.	30 lbs.
Leeward drag - - -	*nil*	*nil*	*nil*

DUCHEMIN'S FORMULA.—This is as follows:—

$$P^N = P\,\frac{2\sin\phi}{1+\sin^2\phi}.$$

Assuming that $P = 30$ lbs., this gives—

Angle of Roof Surface.	30°.	45°.	60°.
Windward pressure, P^N -	24 lbs.	28 lbs.	28 lbs.
Leeward drag - - -	*nil*	*nil*	*nil*

The normal pressure P^N having been determined, its horizontal and vertical components P^h and P^v can

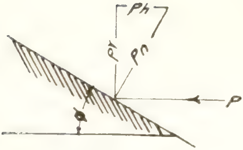


FIG. 146.

easily be obtained graphically, as Fig. 146, or by calculation.

NATIONAL PHYSICAL LABORATORY TESTS.—The National Physical Laboratory experiments, however, demonstrate the necessity, not only of taking into account the leeward drag, but also other conditions, as both the windward pressure and the leeward drag vary according to whether the roof be

(*a*) Mounted on columns through which the wind can pass *harmlessly*.

(*b*) So constructed that a direct internal pressure may be established on the interior by open sides or by large open doors or windows.

(*c*) Erected on buildings, with the internal surface protected from wind pressure.

Dr Stanton has very kindly summarised for the author the voluminous results obtained, as follows:—

Pressure in lbs. P per square foot of roof surface = P K V^2 where V = wind velocity in miles per hour.

Values of K.

ROOFS OF TYPE (*a*).

Angle of Roof Surface.	30°.	45°.	60°.
Windward pressure, P^N	+ 0.0015	+ 0.0028	+ 0.0034
Leeward drag - -	negligible	negligible	negligible

ROOFS OF TYPE (*b*).

Angle of Roof Surface.	30°.	45°.	60°.
Windward pressure, P^N - Leeward drag - -	+ 0.0015 − 0.0022	+ 0.0028 − 0.0027	+ 0.0034 − 0.0032
Total lateral drag on *whole* of roof	0.0038	0.0055	0.0066

ROOFS OF TYPE (*c*) (approximate).

Angle of Roof Surface.	30°.	45°.	60°.
Windward pressure, P^N - Leeward drag - -	negligible − 0.0015	+ 0.001 − 0.001	+ 0.0015 − 0.001
Total lateral drag on *whole* of roof	0.0015	0.002	0.0025

These results applied to a velocity of 97 miles per hour, equivalent to a pressure of 30 lbs. per foot, for comparison with the tabulated results of the Hutton and Duchemin formulæ are as follows:—

ROOFS OF TYPE (*a*).

Angle of Roof Surface.	30°.	45°.	60°.
Windward pressure, P^N - Leeward drag - -	14 lbs. negligible	26 lbs. negligible	32 lbs. negligible
Total - -	14 lbs.	26 lbs.	32 lbs.

ROOFS OF TYPE (*b*).

Angle of Roof Surface.	30°.	45°.	60°.
Windward pressure, P^N	14 lbs.	26 lbs.	32 lbs.
Leeward drag - -	21 ,,	25 ,,	30 ,,
Total - -	35 lbs.	51 lbs.	62 lbs.

ROOFS OF TYPE (*c*) (approximate).

Angle of Roof Surface.	30°.	45°.	60°.
Windward pressure, P^N	negligible	9.4 lbs.	14.0 lbs.
Leeward drag - -	14 lbs.	9.4 ,,	9.4 ,,
Total - -	14 lbs.	18.8 lbs.	23.4 lbs.

It would thus appear that, although the formulæ in common use assume results which are unduly drastic in the case of ordinary (type *c*) roofs, yet they do not adequately express the pressures experienced by roofs of types (*a*) and (*b*), where the wind can cause dangerous eddies on the leeward side, or where it can, to use an old-fashioned term, "get inside the building."

EFFECT OF HEIGHT UPON WIND PRESSURE.—The severity of wind pressure is generally supposed to increase with the height above ground; but the Forth Bridge records show very similar maxima results of considerable differences in altitude. In the design of isolated projections above the roof of a building, such as chimney stacks, towers, etc., allowance should be made for negative leeward drag, for which purpose the values given on page 265 might be increased by about one-third.

CHAPTER XXIV

BRACED COLLAR AND HAMMER-BEAM ROOFS

Fallacious Assumption of Frictionless Joints in Graphic Solutions—Braced Collar and Hammer-Beam Trusses—Outward Spread of Truss Shoes under Load—Stretch of Tie-Rods—Test of Roof Truss for Outward Spread—Stress Diagrams for Hammer-Beam Trusses—Stresses in Hammer Framework—Permissible Stresses in Roof Timbers.

If the fine arched hammer-beam roofs at Eltham, Westminster, Penshurst, or any of the great halls of the fourteenth or fifteenth century had been constructed in the year of grace 1911, it is very improbable that they would have been entrusted to the slender graceful buttresses which have carried them safely for nearly five hundred years. It is still more probable that they would have been provided with rows of horizontal tie-rods, possibly "relieved" in the centre with screw shackles and fussy, meaningless wrought-iron scrolls, effectually ruining the high, uninterrupted vista of arched roof timbers, inducing an uncomfortable feeling of dangerous weakness economically met, and generally recalling the appearance of a bad case of dilapidations temporarily secured.

The modern tendency for excessively massive buttresses and for the introduction of horizontal tie-rods into arched roofs—which ought to be (and generally are) amply efficient without them—may originate with their designers, or it may be the fault of the authorities whose approval has to be obtained. In any case, it is due to a want of appreciation, on the part of one or both, of the true capabilities of such roofs, and can generally be traced to the undiscriminating habit of regarding every problem in roof design as a simple exercise in graphic statics.

FALLACIOUS ASSUMPTION OF FRICTIONLESS JOINTS IN GRAPHIC SOLUTIONS.—The Tudor builder, fortunately for us, lived before the publication of textbooks describing the fascinating labour-saving possibilities of stress diagrams. He based his estimate of the capabilities of his timbers upon practical experience, and was not afraid to trust his constructive instinct. Nowadays, however, it is difficult to imagine any self-respecting draughtsman starting the ½-in. details of a large arched roof until he has drawn, or at least believes he has drawn, a complete stress diagram of the skeleton truss; and he would determine the sizes of all the timbers mainly with regard to the conclusions drawn from the result.

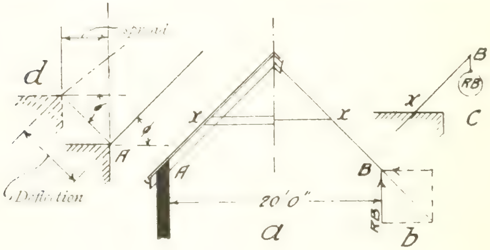


FIG. 147.

Unfortunately, it is very seldom recognised that the usefulness and accuracy of such diagrams are largely discounted by the great fundamental fallacy which underlies all solutions by graphic statics, viz., the erroneous assumption that all intersections of members are frictionless joints, instead of being, as they generally are, particularly stiff and rigid. In fact, in wooden roofs they consist, in almost every case, of continuous unjointed timbers *stiffened* by the connection of another member, and by the addition of bolts and straps.

In iron and steel roofs this fallacy usually results at the most in comparatively insignificant errors on the side of safety, but in arched wooden roofs, if unappreciated, it can result in construction which borders on the ridiculous.

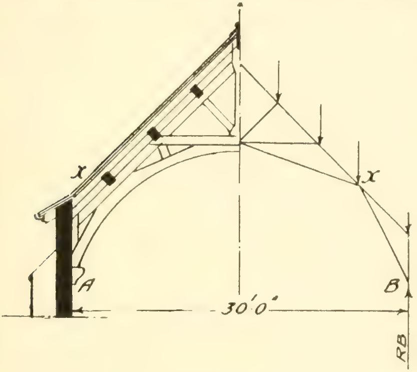


FIG. 148.

BRACED COLLAR AND HAMMER-BEAM TRUSSES.—As will be seen in Figs. 147 to 149, hammer-beam, braced collar and similar roofs are merely elaborations of the simple collared couple shown in Fig. 147 (*a*).

To a draughtsman accustomed to ascertain roof stresses solely by graphic statics applied to a *skeleton* diagram, there is an almost irresistible temptation to assume that because the points $x x$ are apparently unbraced, therefore they must be regarded as flexible, and that there is in consequence a dangerous tendency to push out the walls at A and B—a tendency which must be resisted by heavy walls and buttresses, or by tie-rods, or both. Having assumed the points $x x$ to be flexible, then the logical conclusion follows that

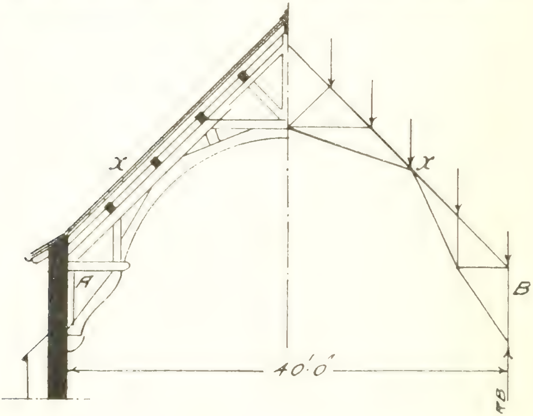


FIG. 149.

the outward thrust must consist of the horizontal component of the compression in x A or x B acting in conjunction with the reaction at A or B, as in Fig. 147 (*b*). In other words, that the truss exerts *as much thrust as it would if the horizontal collar were removed!* Add to this the not unusual assumption of a 50-lb. wind (which is a typical West Indian hurricane capable of levelling probably half the buildings and all the trees in England), and it is not difficult to understand why village halls cost so much on account of the heavily buttressed walls, and why modern church roofs, of types which have stood for centuries, are so frequently disfigured by rows of horizontal ties.

Had the problem, however, been approached with a common-sense regard to the limitations of graphic statics it would have been seen that the points $x x$ are not joints at all, but are the fixed ends of two inclined reversed cantilevers upon which the rest of the roof sits. The nature of the stresses on one of these cantilevers will be more easily seen if it be reversed, as in Fig. 147 (*c*).

OUTWARD SPREAD OF TRUSS SHOES UNDER LOAD.—These cantilevers will, of course, deflect under load and the truss feet A B will consequently open out, *but only as far as that deflection allows them to.* When that deflection is complete they are as rigid under the load as a roof directly braced. It will be found that with roofs properly constructed, with timbers of the ordinary size, this outward spread is extremely small. In fact, it is practically no greater, and may be even less, than the very small outward movement which would be permitted by the elastic stretch in a direct horizontal steel tie under ordinary working stress.

The bending movement due to the reaction at A or B tends to set up fibre stresses at $x x$. The resulting deflection ($\triangle$) of the truss foot in a cantilever of uniform section of length l and depth d under the influence of a fibre stress (f) upon material possessing a modulus of elasticity (E) would be $\frac{2fl^2}{3\mathrm{E}d}$ (see page 192). In timber roofs of ordinary construction the depth (d) would be at least $\frac{l}{8}$, and the extreme fibre stress which would be worked to in determining the section at x would not exceed 10 cwt. per square inch. Taking E at 10,000 cwt. for average fir, the deflection $\triangle$ of the cantilevers Ax and Bx would be—

$$\triangle = l \times \frac{2 \times 10 \times 8}{3 \times 10{,}000} = \text{say } \frac{l}{200}.$$

The length (l) would be about one-quarter of the span A B, so that the deflection of the cantilever would be about $\frac{\text{AB}}{800}$.

It will at once be seen that this can only mean a very small dimension indeed, but it must be still further reduced. In the first place the extreme fibre stress (f) is made up partly by direct compression, which adds no appreciable quota to the deflection. In ordinary cases this would amount to from 5 to 10 per cent., according to the slope of the roof. Secondly, the deflection of a cantilever braced towards the free end, viz., the truss foot, as in larger roofs, Figs. 148 and 149, would be some 25 per cent. less than that of a cantilever of uniform section, as Fig. 147 (a). So that in large roofs the deflection would not exceed say $\frac{\text{AB}}{1000}$.

The outward spread of each truss foot due to the deflection would be almost exactly $\frac{\triangle}{\text{cosec}\phi}$, as in Fig. 147 ($d$). For a roof slope of 30° the deflection must, therefore, be divided by 2 to arrive at the resulting spread; for a slope of 45° by 1.414, and for 60° by 1.154. A timber roof of 40 ft. span at 60° might, therefore, be expected to spread, say, $2 \times \frac{480}{1154}$, or, say, $\frac{3}{4}$ in.

STRETCH OF TIE-RODS.—Now suppose that there were a horizontal steel tie across A B stressed up to, say, 6 tons per square inch. The stretch of such a tie under load would be $\frac{\text{stress per square inch}}{\text{modulus of elasticity}}$, say $\frac{6}{13{,}000}$, or $\frac{1}{2166}$ AB; or in a 40-ft. span $\frac{480}{2166}$, or $\frac{1}{4}$ in.

Therefore not only is the actual spread of an ordinarily well-designed roof of as much as 40 ft. span practically unappreciable, *but a direct horizontal tie would allow almost as much.*

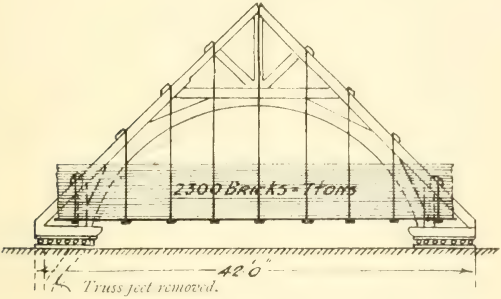


FIG. 150.

Of course, a direct horizontal tie with a tightening shackle affords a ready means of taking up any outward spread, but it must be remembered that quite two-thirds of the deflection would be due to the weight of the roof and covering, which is imposed during construction and is finished by the time the roof is covered in. This can be allowed for by giving the truss feet a little play, or even by erecting them on rollers of gas pipe subsequently grouted in. In any case, it is far too small to warrant the disfigurement and expense of tie-rods. The latter could not be used to take up the occasional deflection due to wind or snow, which should be so small as to be inappreciable.

TEST OF ROOF TRUSS FOR OUTWARD SPREAD.—The author is quite aware of the practical difficulty of convincing local surveyors and other authorities of the truth of the foregoing calculation, in spite of the existence of well-known proofs nearly five hundred years old. But it is not difficult nor expensive to have one truss made, set up on the ground on pipe roller bearings, and loaded with bricks in the manner shown in Fig. 150. The calculated outward spread of this particular hammer-beam truss, of which a photograph is shown in Fig. 151, was 0.63 in. It was loaded with bricks as shown, on a platform of putlogs

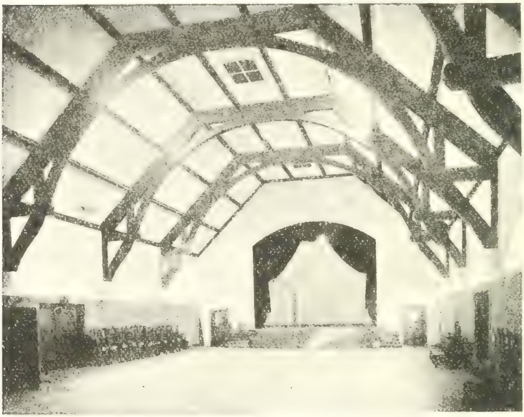


FIG. 151.

and scaffold boards slung from the purlin points, the outward spread being measured with a steel tape. Upon the first application of the load the feet moved outwards slightly over 1 in. on account of the tightening up of the various joints. They came back $\frac{3}{4}$ in.

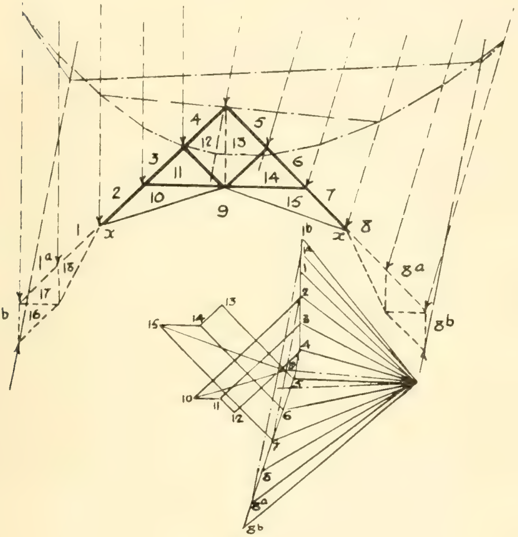


FIG. 152.

on the removal of the load, and spread $\frac{3}{4}$ in. in each of the two reimpositions of it at intervals of two days. When the trusses were erected on brick and stone seats instead of pipe rollers no movement was detected.

It is sometimes suggested that horizontal ties aid the acoustic properties of arched timber roofs; but if the general proportions are correct, and the roof is ceiled at not too high a level, they should not be necessary in order to reduce echo, and they therefore only tend to impair resonance.

The village hall illustrated leaves nothing to be desired as regards absence of reverberation.

Stress Diagrams for Hammer-Beam Trusses.—The stress diagram of a hammer-beam roof becomes quite a simple matter if it be regarded as consisting of three separate portions, as in Fig. 152. It will be found that the ordinary king post tie now becomes a strut under quite a moderate load. If the wind pressure is sufficiently great and the structural load sufficiently small one side of the curved brace might be under compression, but this is an unlikely contingency. The stresses will be much more readily grasped by making a small model of blind laths.

Stresses in Hammer Framework.—The stresses in the parts of the hammer end can be best appreciated by reversing it as shown in Fig. 153. It will be seen that the accuracy of any attempt to calculate them as parts of a jointed frame will be most materially affected by the construction of the joints. With sound joints they are practically indeterminate, and this portion of the roof would probably be designed with the curved braces 18-9 and 16-9 sufficiently strong to transmit, say, half the main reaction to the fixed point X.

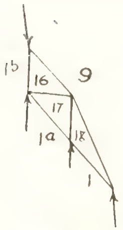


Fig. 153.

Permissible Stresses in Roof Timbers.—The permissible stresses usually adopted for roof timbers are about 6 cwt. per square inch in compression, and 3 cwt. per square inch in tension—the latter being kept low on account of the difficulty of forming joints in small timbers which will pick up tension.

The sizes of arched timbers are determined by the direct load + the effect of eccentricity as before described, but it should be remembered that sharp curves may cut away the continuity of the fibres to a dangerous degree.

PART V

ARCHES AND DOMES

CHAPTER XXV

ARCHES

Theoretical Difficulties in Determining the Stresses in Rigid Arches—Determination by Application of the Principle of Least Work—Stresses in Actual or Assumed Voussoirs—Line of Thrusts—Eccentric Pressure on Voussoir Joints.

THEORETICAL DIFFICULTIES IN DETERMINING THE STRESSES IN RIGID ARCHES.—It is a somewhat striking fact that two of the earliest forms of construction, the rectangular beam and the rigid arch, should still present the most difficult problems in structural mechanics. The reason is that our theoretical ideas are built upon the consideration of known forces acting along defined paths and calling up balancing reactions or internal stresses which also act along definite paths.

When, however, as in the timber beam and the rigid arch, an excess of material offers to the internal stresses the choice of several different paths, each possessing different advantages in leverage, the problem becomes involved, and must be solved either by complex mathematical theory or approximated within the limits fixed by practical considerations.

In skeleton framed structures, such as roofs and lattice girders, the stresses are confined to more or less definite paths and their action along such paths can be easily traced. The same is true of built-up girders and rolled joists with well-defined webs and flanges; for in such it can be assumed without serious inaccuracy that the flanges take the horizontal stresses and the webs deal with the shearing stresses.

But in wooden beams the excess of web in the section affords a number of paths along which the stresses can pass and intermingle; and as a result it is found that rectangular beams bear greater ultimate stresses than are explicable by the simple theory usually adopted. But as in practice this higher ultimate stress is found to apply to all rectangular beams and can be determined experimentally, it is possible to neglect the gap in our knowledge and to design wooden beams with as much confidence as we design rolled joists and built-up girders.

A similar difficulty arises with regard to rigid arches. In a suspended arch of hinged links, such as a suspension bridge or a funicular polygon, the stresses must pass along the links from hinge to hinge, and in a masonry or concrete arch into which hinges are introduced at the springings and at the key, it can be assumed with certainty that the stresses will pass through the pivots at these points, and upon that basis they can be dealt with. But in an ordinary rigid arch the stresses can choose any path they like inside or even outside the arch ring, and it is no easy matter to define precisely what direction they will take.

DETERMINATION BY APPLICATION OF THE "PRINCIPLE OF LEAST WORK." — It should be remembered that structures never do any more work than they can help in resisting the external forces. If a rigid wall be kicked with a 1-lb. kick the internal stresses do not trouble to send back more than a 1-lb. reaction, although they may be quite capable of returning a 50-lb. reaction to a 50-lb. kick. This interesting characteristic, known as the "principle of least work," enables many problems to be solved which would otherwise be indeterminate. The internal stresses called up in a structure by the action of external loads are always the least which will afford the necessary resistance. If, therefore, in any structure under external load a path can be defined, along which the internal stresses can act with the least trouble, they can be relied upon to take that path. If in an arch ring internal stresses could resist the action of external loads along any one of several different directions, they will not go out of their way to choose a more difficult path—where they would have to crush a part here, open a joint there, and induce a bending moment somewhere else — so long as they can find one along which they can act without going to all that trouble. Every rigid arch ring offers to the internal stresses a number of paths of action which all possess equal total advantages in leverage, but all of which (except in very exceptional circumstances) tend to damage the arch more or less. For instance, a certain loading on an arch might be equally well resisted by stresses passing mainly along the top edge or extrados which would tend to crush the top edge and open the soffit joints, or it might be resisted by stresses concentrated along the soffit at one part and outside the extrados at the other, and tending to bodily distort the arch ; or, again, it might be resisted by stresses passing peacefully along near the centre line of the voussoirs where their passage would only create minute bending moments. Under these circumstances they will choose the central path : and under all circumstances they will choose the easiest path they can find.

The first operation in arch design is, therefore, to ascertain the path along which the internal stresses can act *with the least trouble.* When this is found, the extent of those internal stresses can readily be determined.

STRESSES ON ACTUAL OR ASSUMED VOUSSOIRS.—An arch does not necessarily consist of a number of tapered voussoirs spanning an opening. It may consist of half-brick rings or of monolithic concrete, and when stone voussoirs are strongly cemented together the whole becomes practically a monolithic jointless structure. But whatever its construction, it can be divided up into any number of assumed voussoirs, each of which can be considered as a separate structure in equilibrium under the forces acting upon it. Fig. 154 shows typical voussoirs extracted from the haunches of the arch shown in Fig. 155. Upon any voussoir thrust A B is delivered by the action of the load between it and the keystone, and on reaching the voussoir this thrust meets the weight of the voussoir, and any load which it may be carrying (B E) acting vertically downwards, and combines with it to form a new thrust B D which it delivers to the next voussoir. The direction and extent of this new thrust can be ascertained by drawing a parallelogram of forces B C D E, or by the triangle of forces B D E.

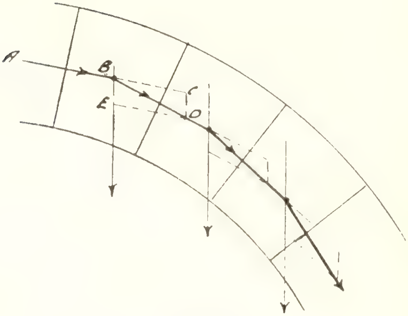


FIG. 154.

The thrusts are, of course, not really concentrated on the lines A B and B D, but are distributed over the voussoir joints. It is, however, more convenient to consider them as acting at their centres of gravity and equally correct, just as the weight of the voussoir can quite correctly be assumed as acting downwards through its centre of gravity.

LINE OF THRUSTS.—Fig. 155 shows the process extended, the lines of thrusts on each voussoir being joined up to form a line of thrusts for the entire

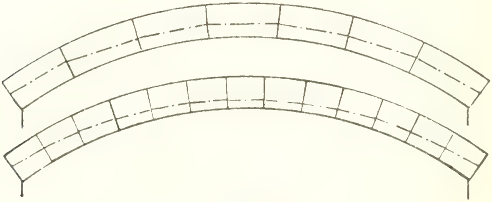


FIG. 155.

arch. It will be seen that the larger the number of assumed voussoirs, the more closely the line of thrusts approximates to a curve.

In drawing the line of thrusts for an entire arch, it is more convenient not to draw the triangles of forces separately on each voussoir, but to combine them in one figure outside the arch, and to transfer on to the arch short lengths of line from voussoir to voussoir parallel to the lines of thrust. The bent or curved line thus obtained shows only how far the general line of arch thrust departs from the centre line of the arch, the actual thrusts being measured on a combined diagram of force triangles. This operation is shown in Fig. 166, Chapter XXVI., the horizontal thrust H at the crown being first calculated in the manner explained later and set out; the vertical weights of voussoirs and load set down vertically, and all the triangles of forces H^1, H^2, etc., drawn at once by joining up the points 1, 2, 3, etc., to point H.

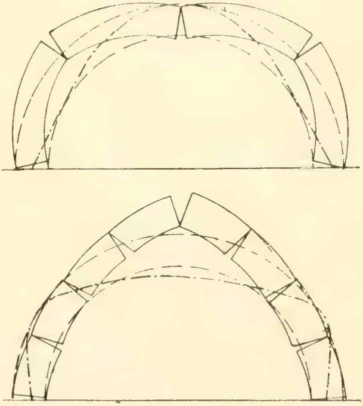


FIG. 156.

If the centre of pressure at a joint does not pass through the centre of the joint there is a tendency to twist the voussoir round bodily and to distort the arch. This is clearly shown in Fig. 156, and in the flexible model, Fig. 157, where uncemented joints can be seen opening when the loading is such that the line of thrusts is close to or outside the ends of the joints.

It is somewhat confusing to note that where the load is greatest and the arch tends to cave in, the line of thrusts tends to *rise*, and it is as well to consider briefly what this line of thrusts really is.

It should be remembered that in a body subjected to tension, such as a link or a link polygon, the particles of that body are called upon to supply equal *and opposite* reactions to the external tensional forces. Similarly they would resist external compressive forces by equal and opposite expansive reactions. A link polygon only comes to rest when the links have automatically arranged themselves in that particular shape which enables the reactions called up from the joints to pass along the centre lines of the links. Then, and only then, is the structure in equilibrium. It is, of course, liable to distortion by the slightest alteration of the external forces and is, therefore, although in equilibrium unstable, whilst the external forces remain unaltered. It is possible to conceive that an arch of links under

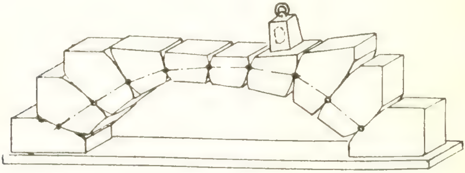


FIG. 157.

compression could be arranged into a similar shape, in which the directions of the internal reactions called for by a given distribution of loads coincided exactly with the direction of the links. There would be no tendency for any part of such a link arch to move, although the instability of its equilibrium is obvious. Its shape would be precisely the same as that which it would assume if the forces acting on its joints were equal but opposite, *i.e.*, if they were pulling upwards instead of pushing downwards. Under such circumstances the links would be highest where the pull is greatest. The shape of a link arch in equilibrium under load is, therefore, that which it would have to assume in order to supply all the necessary equilibrants or reactions to the loading. A line of thrusts is such a line of *reactions*, and represents the directions along which the reactions called up by any loading tend to act.

When these reactions are provided by an arch of some curve different to the line of thrusts, then such an arch would not be in equilibrium and would be liable to distortion at any point where the centre line of the voussoirs failed to correspond with the line of thrusts. The tendency to distortion will be serious or negligible according to the extent of the deviation.

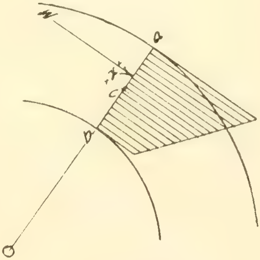


Fig. 158.

Eccentric Pressure on Voussoir Joints.—

When the line of thrusts on any part of an arch ring passes through the centre of the joint $a\,b$, Fig. 158, the stress is equally distributed over the arch ring, and if the area of the voussoir joint in square inches be called A and the thrust W, the intensity of pressure at every point of the joint will be $\frac{W}{A}$ per square inch. If, however, the line of thrusts passes through the arch ring at a distance x from the centre line, then, in addition to the pressure per square inch $\frac{W}{A}$, there is a bending moment $W \times x$, which tends to twist the joint about the centre C, or if it cannot do that to set up compression at a and tension at b. As a and b are already under compression, the result is increased pressure at a and decreased pressure at b, and if the maximum tension stress caused by the bending moment Wx at b be equal to the initial compression $\frac{W}{A}$, then there will be no stress at b, and if it be greater than $\frac{W}{A}$, then b will experience a net tension.

As explained in Chapter XII. the moment of resistance of any section to bending moment is the section modulus (Z) multiplied by a stress per square inch, and the greatest stress per square inch f by which any B.M. in an arch ring will increase the initial compression $\frac{W}{A}$ on one side of the joint, and decrease it on the other, is $\frac{BM}{Z}$. Z in the case of rectangular sections of breadth b and depth d always $= \frac{bd^2}{6}$. Therefore to ascertain the maximum and minimum pressures exerted in an arch ring when the centre of thrust is displaced from the centre of the joint the same simple formula is used as in the case of eccentrically loaded columns and retaining walls, viz., $\frac{W}{A} + \frac{BM}{Z}$ for the maximum compression, and $\frac{W}{A} - \frac{BM}{Z}$ for the minimum compression (which may be a minus quantity, *i.e.*, tension).

The initial compression is reduced below zero on one side when the centre of thrust falls outside the middle third of the arch ring, and in ordinary practice it is customary to make arch rings sufficiently stout to keep the thrust lines within the middle third, or at least the middle half. But the useful little formula $\frac{W}{A} \pm \frac{M}{Z}$ enables the maximum and minimum compression to be calculated wherever the line of thrust may be.

CHAPTER XXVI

ARCHES (*continued*)

Selection of Arch Curve with Regard to Load Distribution—Corbelling of Bonded Masonry over Openings—Concealed Arches in Lintels—Increased Stability Due to Loading—The Design of Arches for Symmetrical Loads—The Design of Arches for Unsymmetrical Loads—Rolling Loads on Arches.

SELECTION OF ARCH CURVE WITH REGARD TO DISTRIBUTION OF LOADING.—In considering what form of arch is most suitable for any given system of loading, it will be found of considerable convenience to set out an approximate line of thrusts. Then the particular form of curve most nearly approximating to the curve of the line of thrusts can be adopted for the arch. For an evenly distributed load obviously a flat segmental curve, which is practically a parabola, will give the most economical results as far as distortion of the arch ring is concerned, although, of course, the flatter the curve the greater will be the thrust both on the abutments and the voussoir joints. For a load mainly concentrated at the apex the pointed Gothic arch (Fig. 161) would probably be the most suitable; whilst for a load increasing from crown to springing the best form would be either a segmental arch with a generous proportion of rise, or a semicircle, according to the rapidity of the increase.

The semicircular arch would appear at first sight to be an extremely strong one, which gives no lateral thrust at the springings. This is very far from being so when such an arch is loaded with a uniform load. In that case the curve of thrusts approximates to a parabola, and is certainly not vertical at the springing.

CORBELLING OF BONDED MASONRY OVER OPENINGS.—Bonded masonry will corbel itself over an ordinary opening as shown in Figs. 159 and 160, but only when the side piers are able to withstand the lateral thrust. This lateral thrust is determined by the bond of the masonry, and when the side piers are of a width less than half the span of the arch it is not customary to trust to the corbelling action to relieve the load on the arch. The arch would then be designed to take the full vertical load above it, whereas when corbelling action can be relied upon

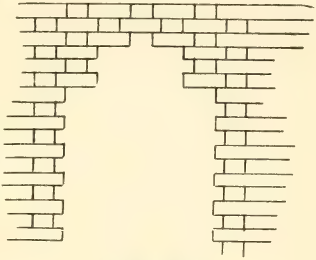
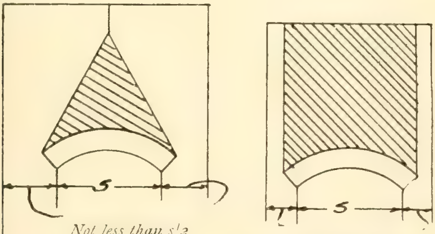


FIG. 159.



Not less than s/2

Less than s/2

FIG. 160.

it need only be designed to take a triangular load which would be best carried by a pointed arch (Fig. 161). In fact it would not be unreasonable to attribute the first introduction of the Gothic arch to an intelligent appreciation of its superiority of form over the Roman semicircular arch for spanning openings in masonry. Structural workmanship declined rapidly after the fall of the Roman Empire, and inferior masonry and poor mortars would quickly disclose the inherent weakness of the semicircular arch when it was deprived of the assistance afforded by the strong cements and excellent workmanship of the Roman masons.

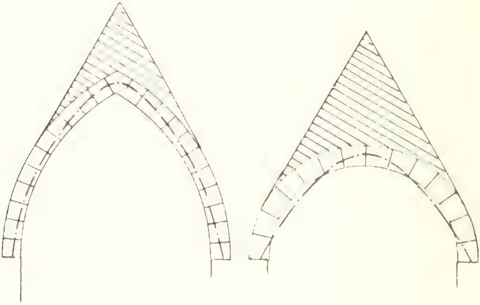


FIG. 161.

CONCEALED ARCHES IN LINTELS.—Concealed arches likewise exist in lintels and flat concrete floors, which are confined or *encastré* at the ends or edges as shown in Fig. 162.

INCREASED STABILITY DUE TO LOADING.—Professor Adams has instanced the case of a half-brick semicircular arch in cement, 10 ft. span, which was intended to form the covering of a small reservoir, but which failed at the haunches under its own weight. Fig. 163 shows such an arch with its line of thrust set out from the centre of the springing and the abutment. Figs. 164, 165 show the same half-brick arch but with the haunches filled in with light earth and broken brick respectively. This increases the amount of the thrusts, but causes them to fall within the arch ring and renders it stable. It will now be easy to realise the reason, as well as the limitation, of the oft-quoted statement that an arch may be

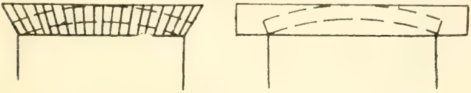


FIG. 162.

stronger loaded than unloaded, which is only the case when the load is not severe enough to crush the ring or push out the abutments. As the load increases, the horizontal and all other pressures increase in amount, although they may be in a more favourable direction as regards distortion of the ring.

THE DESIGN OF ARCHES FOR SYMMETRICAL LOADS.—The curve of arch to be used for any given situation having been decided upon, it is necessary to assume dimensions for the arch ring, and then to ascertain whether it is possible to draw a line of thrusts of the weights thereby involved, which will fall within the limits of deviation fixed upon between the line of thrusts and the centres of voussoir joints. If this cannot be done then a deeper arch ring must be tried.

The limits of deviation are usually the middle half of the arch ring for arches loaded with bonded masonry whose joints might be expected to resist

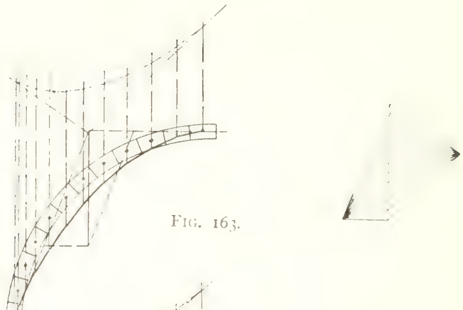


FIG. 163.

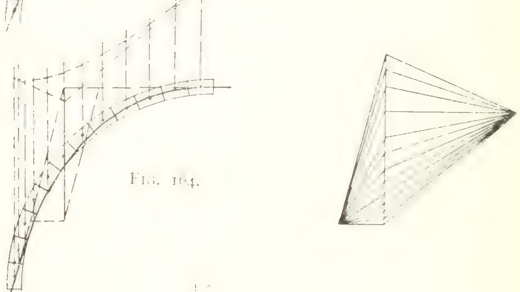


FIG. 164.

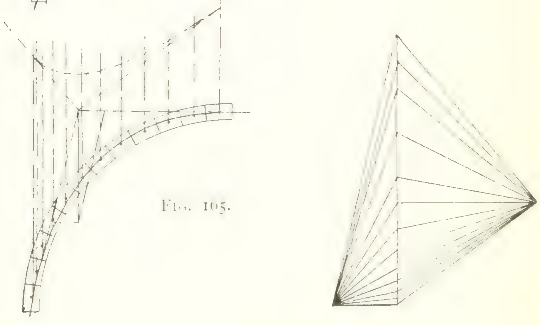


FIG. 165.

distortion; and in the cases of arches carrying light filling only, the middle third.

If an arch of paper or cardboard be made and loaded at one point by pressure of the finger, it will be found that the resulting distortion varies from springing to springing, but is equal across the arch, so that any short width (say 1 ft.) of wide arch may be taken as a measure of the whole, and concentrated loads may be assumed to be distributed over the whole width of arch.

DETERMINATION OF HORIZONTAL THRUST.—When considering a series of vertical loads as being hung on a series of links, there is an infinite number of link systems to choose from, each one of which would give a different horizontal stress at the apex. Everything connected with a link polygon depends upon the polar distance representing the horizontal stress, without which the delineation of the polygon cannot be commenced. Similarly, in setting out a line of thrusts the first operation is to determine the horizontal crown thrust. In symmetrically loaded arches, the thrust will be horizontal at the centre and is calculated as follows:—

In Fig. 166 let A B C D represent half of the arch, and A F E D represent the superincumbent load, and W G a line passing through the centre of gravity of the two.

If a horizontal thrust be assumed acting as at D C, and an inclined resultant on the skewback acting at I, then the half arch will be kept in equilibrium about the point I by—

1. The total weight of arch and loading W, acting at a leverage y in a clockwise direction.

2. The horizontal thrust acting at H in an anticlockwise direction with the leverage X.

These two opposing moments must, therefore, be equal to each other, *i.e.*,

$$W \times Y = H \times X, \text{ or } H = W \times \frac{Y}{X}.$$

The horizontal and vertical forces being known, the inclined resultant on the skewback can be obtained from a parallelogram of forces as shown. This inclined resultant is also equal to the steepest line in the force diagram.

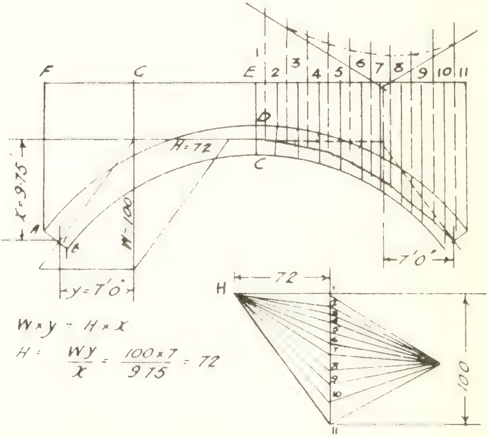


FIG. 166.

If the assumed position of H and I at the ends of the lines of thrusts be correct, the line can now be traced. But it is obvious that either end of the line can start anywhere on the arch ring, and that any alteration in their positions alters the leverages x and y, and consequently the horizontal and inclined thrust.

A line of thrusts is therefore drawn from the centres of crown and skewback, and if it falls within the permissible limits of deviation, and the pressures are not too great for the material, the calculations are finished. If it does not, then the points of leverage are shifted to the particular extremities of the limits of deviation at crown and springing, which will best tend to correct the error. For instance if a line of thrusts drawn between the centre of the section at the springing and the centre of the section at the crown passes outside the top of the middle third at the haunches, the horizontal thrust would be recalculated with the leverages measured from the lower limits of the middle third at springing and crown instead of from the centres of the section. A fresh line of thrusts is then drawn, and if this also passes the limits of deviation then the arch ring must be increased or altered in shape.

There is a well-known method devised by Professor Fuller and modified by Professor Perry which avoids the necessity of drawing more than one line of thrusts. A description of this will be found in Rivington's "Building Construction," vol. iv. The author has doubts as to whether this short cut, which is somewhat involved and intricate, does not take longer than drawing a couple of thrust lines. The latter method is certainly more clear and straightforward, and has the advantage that one can calculate the maximum and minimum pressures on the arch ring at any point of maximum deviation.

THE DESIGN OF ARCHES FOR UNSYMMETRICAL LOADING.—The most convenient method of dealing with an arch under irregular load is to divide the extrados up into equal horizontal widths or panels, draw voussoir lines, and ascertain the total load on each horizontal panel.

It should be recollected that when a concentrated load acts through earth filling or masonry it is more or less spread—the angle of spreading being usually taken at 60° unless it acts through masonry so bonded as to permit of a greater angle.

Precisely the same procedure is then adopted as with symmetrical loading; but before the horizontal and inclined thrusts can be calculated, it is necessary to find out where the thrust will be horizontal. This may be under the centre of gravity of the loads and probably will be. Its position would be at the apex of any funicular polygon corresponding to the particular loading. All that is necessary, therefore, is to draw a steep funicular polygon of the loads; and the polygon will be steep if the polar distance chosen is small. Then the thrusts will be horizontal at the point where the polygon is deepest, and this point is the virtual crown of the arch. Two funicular polygons are then necessary to establish the position of the centre of gravity lines of the two portions of the loading lying on each side of the virtual crown. These being determined, the horizontal thrust is calculated and should be the same whichever side it is calculated from, the necessary leverages being taken from the centre of the arch ring at springing and virtual crown. If these conditions do not give satisfactory results the points of leverage are shifted as before, and if necessary the arch ring is altered.

ROLLING LOADS ON ARCHES.—Rolling loads generally tax the stability of an arch more severely by distorting the line of thrusts than by increasing the measures. The effect on the line of thrusts caused by the moving load in various positions can readily be ascertained by the funicular polygon.

CHAPTER XXVII

DOMES

Investigation of Stresses in Loosely Jointed Frames—Keystones Unnecessary in Domes—Hoop Stresses—Radial Thrust—Stresses in Thin Hemispherical Dome—Stresses Due to Weight of Material Independent of Thickness—The Elimination of Hoop Tensions.

INVESTIGATION OF STRESSES IN LOOSELY JOINTED FRAMES.—The nature of the stresses in domes can best be appreciated after studying the effect of various loadings on loosely jointed model-framed sections with bars at various degrees of inclination.

In the triangular roof couple frame shown in Fig. 167 the compressions in the rafters and the tension in the tie will be found to vary with the load only if the angle of inclination of the rafters remains constant. Even a slight difference in the angle at which the rafters are presented to their work of holding up the central weight will very materially affect the stresses in them and in the horizontal tie. The same is found to be the case

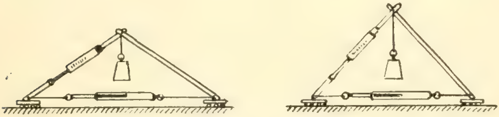


FIG. 167.

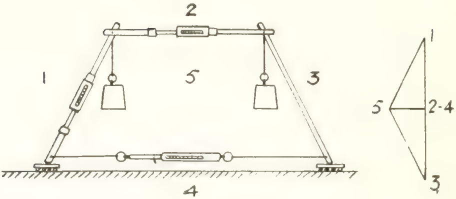
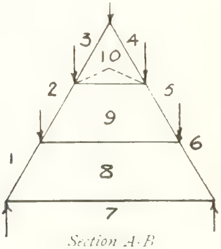
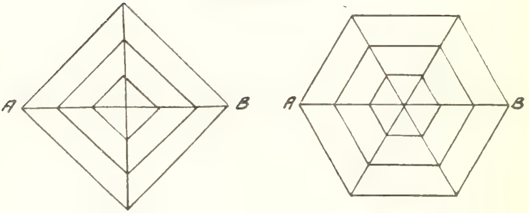


Fig. 168.

with the frame shown in Fig. 168. Experiments with the frame shown in Fig. 169 are also very instructive. It will be found that however great a load is put on the apex, it is impossible to throw



Section A-B



Plan *Plan*

Fig. 169.

the bar 10-9 into tension so long as 4-10 and 5-9, and 3-10 and 2-9 are in line, and there are any loads at 2-3 and 4-5. This is what would be expected from an examination of the stress diagrams, which also show that as far as the horizontal bars above the bottom bar are concerned, the effect of each alteration in load is peculiarly local—an alteration in the apex load does not affect the bars 8-9 and 9-10, nor does any alteration in the loads 2-3 and 4-5 affect the stress in 8-9, but all alterations affect the stresses in the *inclined* bars and the *bottom horizontal* bar.

When, however, the *inclination* of bars 3-10 and 4-10 is altered as shown by dotted lines, the stresses may be quite different. The stresses in bar 9-10 which previously was *always* compression, whatever the load, may now be reduced to zero or even turned into tension according to the apex load; but the stress in bar 9-8 will remain compression.

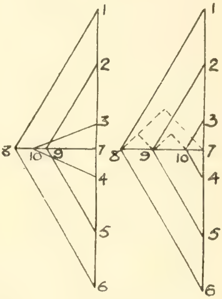


FIG. 170.

Each of these frames may be considered as a section through the angles of a skeleton frame dome which may be square, triangular, or polygonal on plan. The stresses in each horizontal bar may be considered as resolved into two directions along the horizontal purlins of the frames, and the amount of such horizontal stresses can be obtained by setting up in the stress diagrams lines, shown dotted, parallel to the inclination of the sides on plan.

Figs. 172 and 176 show half sections through one of the outer angles of a polygonal dome of

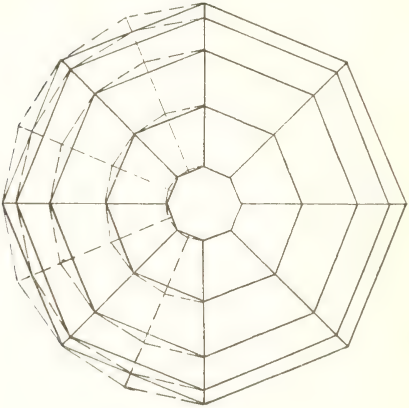


FIG. 171.

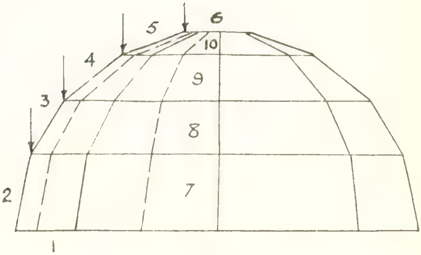


FIG. 172.

eight or sixteen sides, Fig. 171—Figs. 173, 174, and 175 being the corresponding stress diagrams. It will be seen that with a load equally distributed

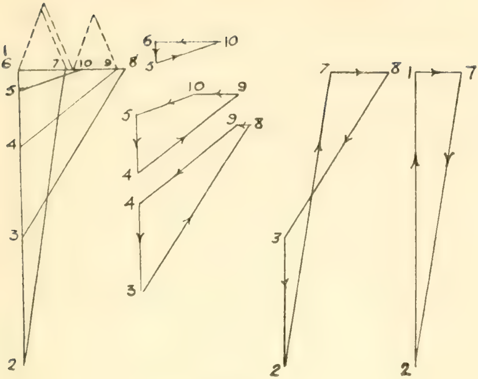


FIG. 173.

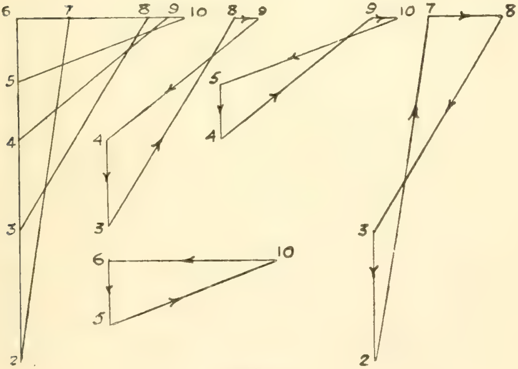


FIG. 174.

over the surface of the frame, Fig. 172, the stress in bars 10-9 and 9-8 measured on the stress diagram, Fig. 173, from right to left, indicate compression, whereas the stresses in bars 8-7 and 7-1 are tension.

If, however, the load 5-6 be increased, Fig. 174, the top bar 6-10 is the only one in compression, and all those below are in tension.

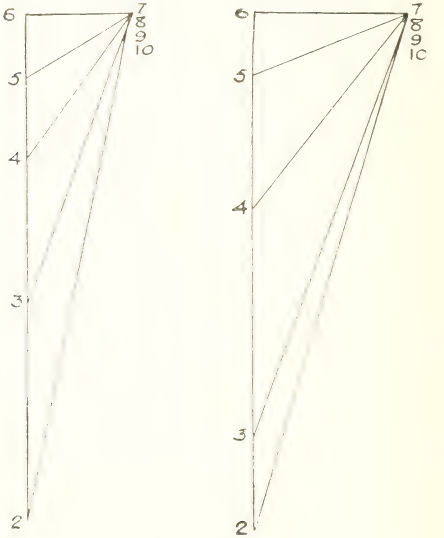


Fig. 175.

Figs. 175 and 176 show that the loads can be so adjusted that all the horizontal bars except the top and bottom are relieved from all stress, and also that for a given loading the same result can be obtained by varying the inclination of the bars.

Let the stresses in the horizontal bars of the frame section, Figs. 172 and 173, be considered as resolved into the hoops of horizontal bars in the complete framed structure. It will then be seen that under an equally distributed load the two bottom hoops, 8-7 and 7-1, are in tension, and the three top hoops, 8-9, 9-10, and 10-6 in compression, in other words that the two bottom hoops are preventing the dome from bursting outwards, and the top hoops are preventing it from collapsing inwards. If, however, the top of the dome be loaded to a sufficient extent, as with a heavy stone lantern, all the hoop stresses below the top ring are tensional, *i.e.*, the dome tends to burst outwards anywhere below the top ring.

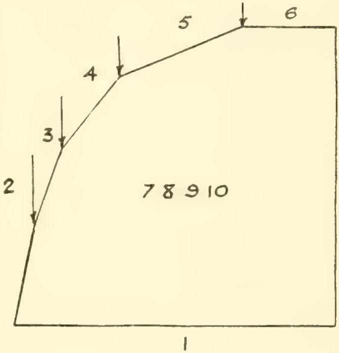


FIG. 176.

As the number of sides of the polygonal dome and the number of slopes in the section are increased the framed structure approaches more closely to the form of a complete or partial hemisphere, and it is not difficult to imagine a hemispherical dome consisting of an infinite number of ribs, but otherwise similar to the skeleton shown in Fig. 172—the number of hoops being, of course, correspondingly increased.

It is evident that by suitably varying the loads such a dome could be relieved of all hoop stresses, except in the top and bottom rings, and also that for any given system of loads there is a certain curve of dome in which no hoop stresses would exist except at the top and bottom rings. It will also be seen that in a straight-sided conical dome (of a section similar to A B, Fig. 169) no *tensional* hoop stresses can occur whatever the apex load, so that such a conical dome would carry a heavy apex load, like a stone lantern, without any tendency to burst outwards above the bottom ring; but in domes with curved sides the effect of a heavy apex load is *to increase the "bursting area" over which the surface of the dome tends to open outwards.* The reason of the triple shells forming the dome of St Paul's Cathedral will now be apparent. The outer shell is designed to a steep curve, best suited to form a conspicuous dominating external feature, which would have been impossible with the flat curve, best adapted to secure internal effect. Both of these forms, however, are unsuitable for carrying the large apex load of the stone lantern, and for this is provided the central cone or funnel-shaped shell, a shape excellent for weight carrying, but suitable for neither internal nor external effect.

KEYSTONES UNNECESSARY IN DOMES.—The essential difference between the arch and the dome can readily be traced in the frame section.

In Fig. 169 the inclined bars above the horizontal bar 9-10 are not essential to the stability of the frame. Similarly, in Fig. 172 it will be seen that any horizontal bar enables the part of the frame below it to act as a complete structure. The horizontal bars in such frames supply, therefore, the place of the horizontal crown thrust which is essential to the stability of the arch. If an arch were provided with horizontal struts, each of such struts would take up any horizontal inward thrust from the portion below it, and enable the portion above it to be removed. The stresses in the imaginary horizontal bars of the dome sections really represent the horizontal hoop stresses—tensile or compressive—which act horizontally round the complete dome. Therefore a dome (provided that its material is capable of resisting horizontally both compression and tension) acts like a tied and strutted arch, which would have to provide no horizontal crown thrust because the horizontal ties and struts would prevent any such thrust from forming.

For this reason such a dome would have no thrust at the crown, because the material of each horizontal hoop provides its own tension or compression, and any portion of a dome can be sliced off without affecting the stability of the remaining portion below it. Thus it is that eyes of any diameter can be left in the crown of a dome, whether it be the domed top of a well or the lantern of a cathedral, although the keystone of an *arch* is a most essential part.

HOOP STRESSES.—If, however, the material of the dome be incapable of resisting tension, and thus unable to offer any resistance to the hoop tension at or above the springing, then the dome will tend to spread out and split from the springing into a series of arched ribs. These ribs would act as ordinary arches, and if correctly designed as such, would be quite stable. Many domes are, in fact, built in the form of arched ribs between the springing and a circular compression ring near the apex—the spandrels being filled in with arches spanning from rib to rib.

The dome of Ste. Sophia, for instance, has openings all round the springings, extending far up the vault. The ribs between these openings act as true arches, and the thrust at the springing is resisted by the counter-thrust of semi-domes and masonry.

The hoop compressions in the upper part of a dome and the hoop tensions in the lower part are of great importance. Their calculation is quite simple when they are regarded properly.

RADIAL THRUST.—The first necessity is to clearly understand what hoop stress is set up by any given outward radial thrust in a circle. In a polygon this can easily be determined. In a circle it is arrived at as follows :—

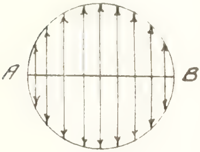


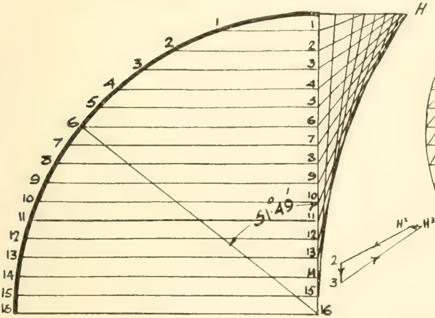
FIG. 177.

If a cylindrical steam boiler, Fig. 177, be subjected to the outward radial bursting pressure of the steam inside it, the tendency to burst would be the same anywhere along the cylinder ; and the strength of the latter can, therefore, be considered one foot or one inch of its length at a time. The total radial thrust per inch run of cylinder would be the steam pressure per square inch $(p) \times$ the circumference. But in order to cause tension in the shell the steam pressures would only be effective in so far as they are able to act back to back along any diameter A B. Their effect would, under these circumstances, be $p \times$ A B, and as their tendency to burst the hoops is resisted by material at two points A and B, the tension which they can exert at any one point in the hoop is $p \times$ radius.

The relation between the total outward radial thrust ($p \times$ circumference) and the resulting hoop tension ($p \times$ radius) is as the circumference of a circle to its radius. The hoop tension T due to a total radial thrust R would be—

$$T = R \times \frac{\text{radius}}{\text{circumference}}.$$

The radius of any circle divided by its circumference is, of course, a constant quantity, $\frac{1}{6.2832}$ nearly. Therefore the hoop tension T called into



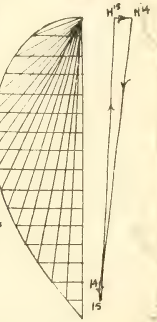


FIG. 178.

FIG. 179.

being by any horizontal radial thrust in a horizontal hoop has only to be divided by 6.2832 to obtain the hoop tension.

This is all the arithmetical calculation needed, everything else can be ascertained graphically.

STRESSES IN A THIN HEMISPHERICAL DOME.—Fig. 178 shows a section through a thin hemispherical dome formed of material equally capable of resisting tension or compression, of uniform thickness, and uniform weight per foot. The section is drawn to such a linear scale that the radius also represents the total weight of the dome to a scale of loads. If the vertical radius be divided into any number of parts as 1-16, and horizontal lines be drawn cutting the dome and section, then the length of each part, 1-2, 2-3, etc., will represent the weight of the complete hoop of dome lying between the corresponding horizontal lines, because the area of any segment of a sphere is directly proportional to its height. In thin domes the pressures may be considered as distributed uniformly over any section and acting tangentially to its surface. From each division on the load line draw lines parallel to tangents to the curve of the hemisphere, viz., at right angles to radii 1-16, 2-16, etc., and meeting a horizontal line drawn through the crown. The resulting diagram is merely a more complete edition of a stress diagram similar to Figs. 173, 174. The lengths cut off the horizontal line between any two inclined lines as H^2 H^3 give the horizontal radial thrusts exerted by the weight of each segmental ring as 2-3, and according as to whether they are measured from right to left or from left to right they indicate compression or tension. For instance, the detached figure of the forces acting at point 2 show by the arrow heads that the horizontal force called for is *compression*, whereas a detached figure of the forces at point 15 would show that the horizontal force called for there is *tension*. In a thin uniform hemispherical dome, loaded only with its own weight, the hoop stresses change from compression to tension at an angle of 51° 49′ from the vertical, viz., at the point where the curve sensibly departs from that of a parabola. Above that point the dome would be stable if built of material possessing no tensile strength ; below it every hoop and every particle is subject to tensile stresses tending to burst it open, and of an intensity equal to the horizontal pressure measured on the diagram ÷ 6.2832. The length of the inclined lines of the diagram give the total downward thrust on each hoop acting between springing and crown due to the weight above it, and to obtain the compression per square foot in the material of the dome caused by the downward thrust, as distinct from that caused by hoop stress, this downward thrust must be divided by the circumference of the ring × the thickness of the material.

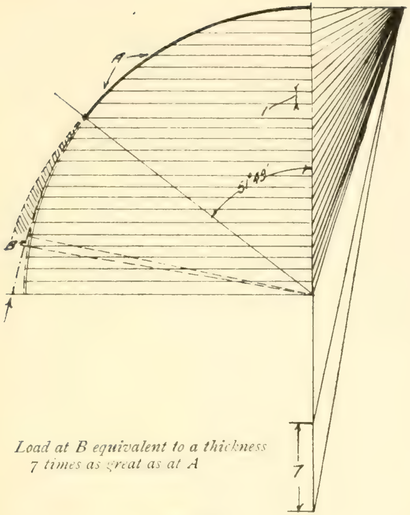
The diagram is more easily measured from if drawn in the method proposed by Mr Dunn (*vide* his paper on "Domes," *R.I.B.A. Journal*, 21st May 1904), as in Fig. 179, in which the radial thrust in each ring is shown by the difference in length between the horizontal lines at the top and bottom of each ring.

STRESSES DUE TO WEIGHT OF MATERIAL INDEPENDENT OF THICKNESS.—It will now be seen that if a dome supports nothing but its own weight, and if it be of material capable of resisting tension, there is no advantage in making it any thicker than is necessary to keep out the weather and resist local loads and wind pressure — for every particle of material added in thickness means so much more load, and the pressure per square foot on the material due to its own weight is exactly the same in a thick as in a thin shell. The part above the 51° 49′ should, therefore, always be as thin and light as possible.

THE ELIMINATION OF HOOP TENSIONS.—Between the bottom ring and the zero ring the local tendency to burst can be neutralised in a dome of uniform thickness either by using material capable of exerting tension, or by reinforcement with tensional steel rings. If the material can neither be trusted in tension nor be reinforced, then this "bursting area" must be treated as consisting of a number of parallel arch rings side by side. Such arch rings would be quite stable and would properly support the upper part of the dome if the line of thrusts were kept within the middle third of the section, and the thrusts are not too much for the material.

The difference between the slopes of the different individual rings of what may be called the *compression area* above 51° 49′ causes an individual hoop compression to each of these rings. At this point the curve of the hemisphere commences to sensibly depart from the curve of the thrust diagram—viz., a parabola—and in consequence hoop tensions begin to appear. The hoop tension of each ring may, of course, be neutralised by the tenacity of the ring, resulting in a hemispherical dome with no outward thrust at the base similar to the frames in Figs. 167 and 168. When the material of the bursting area has no hoop tensional value it can only receive the thrust at the zero ring and deliver it to the springing in the same way that a masonry arch receives and transmits the crown thrust.

The "bursting area" of the dome then consists of an arch circular on plan, and the line of thrusts of such an arch should, in order to neutralise hoop tensions, be kept within the middle third, either by varying its curve or by adjusting the loads by thickening the material (see Fig. 175). Where the load consists only of the weight of the material of the dome the hoop tensions would disappear below the zero ring (51° 49′) if the portion below that



Load at B equivalent to a thickness 7 times as great as at A

FIG. 180.

ring followed a parabolic instead of a segmental contour. They would similarly be eliminated by a gradual thickening of the material towards the springing, as in Fig. 180.

CHAPTER XXVIII

DOMES (*continued*)

Variation of Stresses in Structures due to Differences of Scale—Calculations for Segmental Dome 60 ft. Span—Danger of Concrete made from Coal Residues—Calculations for Hemispherical Dome 100 ft. Span Carrying 120-ton Lantern—Examples of Thin Domes.

VARIATION OF STRESSES IN STRUCTURES DUE TO DIFFERENCES OF SCALE.—When considering the subject of domes it is difficult to avoid drawing comparisons from small everyday objects. For instance, a domed glass lamp shade easily supports its own weight together with that of a heavy additional load without collapsing, in spite of being only a thin sheet of heavy and brittle material, and one is apt to conclude that, making due allowance for relative strengths, the same proportions might be adopted with safety on a much larger scale in domes constructed of lighter material, like well-bonded brickwork reinforced with hoop iron. Although this deduction is not seriously incorrect it must always be remembered that proportions which may be ample in a small scale structure are not *necessarily* even safe in one constructed to a larger scale. A slab of any material 1 ft. × 1 ft. and weighing 12 lbs. per foot super., supported on all its edges, presses on the bearing edges with a stress of $\frac{12}{4}$ or 3 lbs. per foot run. If, however, the dimensions of the slab are doubled the pressure on the bearings rises to 6 lbs. per foot run. If the thickness be also doubled it amounts to 12 lbs. per foot run. If the slab were made to a scale ten times as large, the pressure on the bearings would be 300 lbs. per foot run. The length of the bearings (*i.e.*, the length of the sides of the slab) increases directly with the increase in dimensions, but the weight of the slab increases as the *cube* of the increase in dimensions, a very much quicker ratio.

The difference between operations on a small scale and those on a large scale is thus a very important one. A striking illustration can be seen in any stream with a stony bottom. The small pebbles are rolled over easily, but larger ones remain unmoved, for although the surface which they present to the stream force increases as the *square* of their dimensions, their weight increases as the *cube*.

In a small hemispherical dome the upper 10°, a portion resembling the lid of a round china teapot, would press downwards on the ring immediately below it, and exert a certain pressure per square foot on the material of that ring. In a dome of ten times the size, and of the same relative thickness, the pressure *per foot super.* will be ten times as great. That part of a hemispherical dome which lies above the rings subject to hoop tension, is subject to compression only, acting in directions which can be best described by borrowing a geographical term, viz., latitudinal or hoop stresses and longitudinal stresses from apex to springing. For any given diameter of dome both the latitudinal and the longitudinal compressions per square foot are the same whatever the thickness—a fact which calls for as thin a shell as possible—yet at the same time the stresses vary not directly with the diameter but with the *square* of the diameter, so that the pressure per square foot of material is greater in large domes than in small ones. In the case, however, of concrete and brick domes *loaded only with the weight of their own material*, no ordinary span will give rise to compressive stresses of anything like the full safe crushing stress of the material.

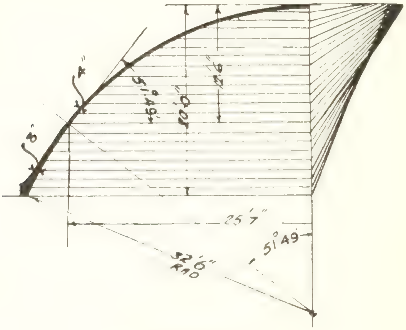


FIG. 181.

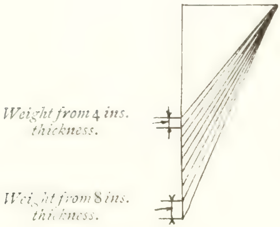


FIG. 182.

CALCULATIONS FOR A SEGMENTAL DOME 60 FEET SPAN.—Take the case of a dome 32 ft. 6 in. radius with a maximum diameter at the base of 60 ft. and a rise of 20 ft. (Figs. 181 and 182). The greatest stress on the material would probably occur at the ring of zero hoop stress, because below this ring the thickness would probably be increased. The diameter of this ring would be the radius of the dome curve (R) × 2 × sin 51° 49′

$$= R \times 2 \times 0.786 = 51.2 \text{ ft.},$$

and its circumference would be $51.2 \times \pi = 161$ ft. The vertical height above the zero ring would be R × ver. sin 51° 49′

$$= 32.5 \times .382 = 12.5 \text{ ft.}$$

The surface of the dome above the zero ring would be $2\pi R \times 12.5$

$$= 6.28 \times 32.5 \times 12.5 = 2{,}545 \text{ sq. ft.}$$

The area of 2,545 sq. ft. is loaded with—

(*a*) The weight of its own material.

(*b*) The weight of the water-tight covering.

(*c*) Wind pressure on one side.

The amount of horizontal wind pressure which a saucer dome can absorb and pass on to its base would be very small, but making the large allowance of 28 lbs. or $\frac{1}{4}$ cwt. per foot super. for asphalt and for heavy wind or deep snow (the two cannot exist together), the vertical load due to (*b*) and (*c*) on the zero ring would be—

$$\frac{2545}{4} = 636.4 \text{ cwt.}$$

Assuming the dome were constructed of ballast concrete in cement and that a thickness of 4 in. would support any local accidental loads, then the weight on the zero ring due to (*a*) would be, at 1 cwt. per foot cube of concrete,

$$\frac{2545}{3} = 848 \text{ cwt.}$$

The weight of this upper segment must be considered as presented to the zero ring in a direction tangential to the curve of the dome at that ring, viz., at an angle of 51° 49′ with the horizon. So that the relation of vertical weight to the inclined thrust which it causes will be in the relation of 1 : cosec 51° 49′ or 1 : 1.275.

The inclined thrust which the zero ring is called upon to support is, therefore—

$$(637 \text{ cwt.} + 848 \text{ cwt.}) \times 1.275.$$
$$1{,}485 \times 1.275 = 1{,}900 \text{ cwt.}$$

If the loads had been correctly set up this could, of course, have been scaled from the diagram without calculation.

The area of supporting surface offered by the zero ring is 161 ft. × 4 in. = $53.\dot{6}$ super. ft.

The resulting pressure per square foot would be $\frac{1900}{53.\dot{6}} = 3.55$ cwt. per foot super. If the thickness of concrete had been 6 in. the pressure per foot super. due to the weight of concrete would have remained the same, but the pressure due to wind and snow would have been distributed over the greater thickness, and the total pressure would have been less, thus—

$$\frac{(637 + 1278) \times 1.275}{80.5} = 3 \text{ cwt. per foot super.}$$

Both of these pressures are far below the safe stress on good concrete.

In that portion of the dome under consideration which lies below the zero ring, the hoop tensions can be eliminated either by reinforcement, by altering the curve to a parabola, or by increasing the thickness.

The necessary alteration of the curve is too slight to be shown, and even when drawn out to a large scale it is very small, although in a more nearly complete hemisphere it would, of course, be much more marked.

To ascertain the increase of thickness necessary to similarly eliminate the hoop tensions over the bursting area, vector lines are drawn, Fig. 182, parallel to the small portions of the curve of each ring; these cut off in the load line lengths corresponding to the necessary increase in thickness.

The resulting section appears to be almost ridiculously thin, especially when compared with those of any ordinary modern dome; but the most elementary experience tells us that the upper part of a dome can be entirely omitted. Obviously, therefore, the less there is of it the better for the lower part. Equally obvious is it that if we use a material which cannot be trusted in tension and do not reinforce it, and then in addition ignore the help given by the hoop *compressions*, designing the upper part as if it were an arch starting with a thick crown, we are forced into providing gigantic lower rings.*

If the material had been 6 in. of breeze concrete weighing 70 lbs. per foot cube, the pressure per square foot on the zero ring would have been reduced to—

$$\frac{\left[637+\left(1278\times\frac{70}{112}\right)\right]\times 1.275}{80.5}=2.27 \text{ cwt. per foot super.}$$

---

* The flat saucer domes, 60′ 0″ diameter, at the Roman Catholic Cathedral at Westminster are 1′ 1″ thick at the crown and 3′ 0″ thick at the springing. It would appear that their enormous weight sensibly affected the design.—(*Vide Trans. Architectural Association*, vol. xxii. (1907), pp. 121-123).

DANGER OF CONCRETE MADE FROM COAL RESIDUES.—Concrete made from coal residues is apt to contain sulphur ingredients which rapidly deteriorate steel. Pumice would be preferable if it were desired to construct a very light dome with steel reinforcement.

CALCULATIONS FOR DOME 100 FEET SPAN CARRYING A 120-TON LANTERN.—Fig. 183 shows a very large, thin, and light concrete dome carrying a heavy lantern. Assuming this dome to be 100 ft. diameter, 7 in. thick of pumice concrete, and carrying a lantern 20 ft. diameter weighing 120 tons, the stresses involved will be as follows:—

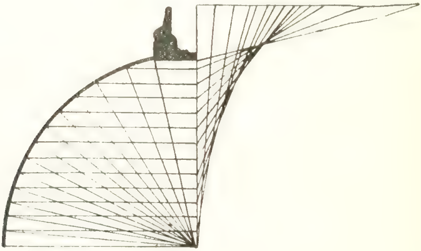


FIG. 183.

Allowing 28 lbs. per foot for wind pressure and covering, and 70 lbs. per foot cube as the weight of pumice concrete, the weights would be—

$$\text{Area of dome} = 2\pi r^2.$$

Weight per foot super. = 35 lbs. concrete + 28 lbs. wind and covering. Weight of shell—

$$\frac{2 \times 3.14 \times 50 \times 50 \times 63 \text{ lbs.}}{2240} = 432 \text{ tons.}$$

Shear on ring at top due to lantern—

$$\frac{120 \text{ tons}}{20 \times 3.14 \times .5} = 3.8 \text{ tons per foot super.} = 59 \text{ lbs. per sq. in.}$$

Longitudinal thrust on ring at top, measured from diagram—

$$\frac{480}{20 \times 3.14 \times .5} = 15 \text{ tons per foot super.} = 215 \text{ lbs. per sq. in.}$$

Longitudinal thrust on bottom ring—

$$\frac{432 + 120}{100 \times 3.14 \times 0.5} = 3.5 \text{ tons per foot super.}$$

The weight of shell is set out as shown in Fig. 183, and divided into convenient lengths to represent feet or multiples of a foot, and the weight of the lantern is superimposed.

It will be seen that the hoop tensions occur right up to the lantern. The greatest radial thrust in any ring 1 ft. high is 8 tons, which divided by 6.28 gives a bursting pressure of 1.27 tons. This would require an area of steel stressed to $6\frac{1}{2}$ tons per square inch of .195 sq. in. which would be provided by two rods of $\frac{3}{8}$ in. diameter every 12 in. So that even with such an extremely thin light dome shell as this, loaded at the apex to such a great extent, the reinforcement necessary to resist the hoop tensions would be provided by a net of $\frac{3}{8}$ in. latitudinal bars 6 in. apart, with, say, $\frac{1}{8}$ in. longitudinal wires 2 ft. apart to keep them in place. The shearing and direct stress on the concrete immediately below the lantern, although possibly not too great for dead loads, would also require to be met by reinforcement when increased by wind pressure on the lantern.

EXAMPLES OF THIN DOMES.—Thin domes are not merely an impracticable deduction from complicated theory, they have been built, and support great weights.

Some remarkable examples exist in Spain of very thin domes of large span—probably Roman work—formed of well-bonded tiles in cement. Similar domes have been erected by the Guastavina Company in America with shells of 50 ft. and 60 ft. in

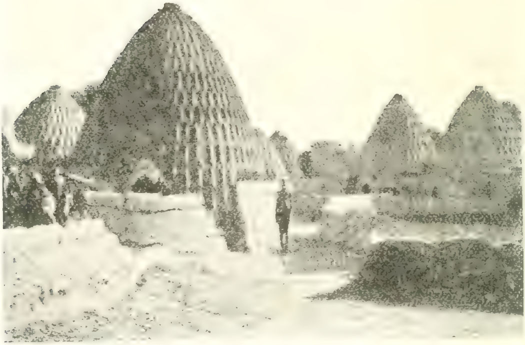


FIG. 183*a*.

diameter, and 3 in. to 4 in. thick, carrying lanterns at the apex of more than their own weight. Illustrated descriptions will be found in vol. xxiii. (1908) of the *Architectural Review.* In these domes the grip of the bonding of the tiles is relied upon to supply the hoop tension. But as domes are liable to cracks from unequal settlement of the bearings it would be safer to provide against these stresses by some form of steel reinforcement, or to neutralise them by slightly altering the curve of the dome or the thickness of the shell.

Very striking evidence of the stability of extremely thin domes when constructed to the same curve as the line of thrusts is also to be found in the mud huts of the Central African natives. The author is indebted to the publishers of Miss Olive Macleod's book, "Across Lake Chad,"* and to that lady, for

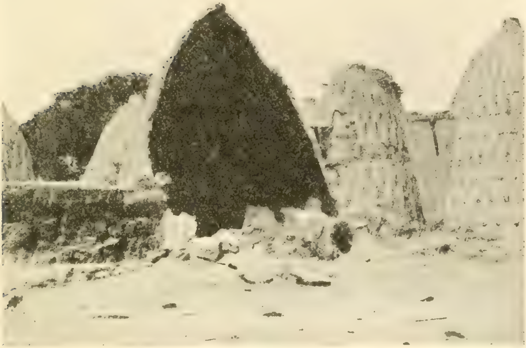


FIG. 183*b*.

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* "Across Lake Chad through British, French, and German Territories," published by Mr Wm. Blackwood, George Street, Edinburgh.

permission to reproduce the photographs, Fig. 183, *a* and *b*, taken by her at Musqum in the German Kameruns. These remarkable examples of traditional structural skill are circular huts about 20 ft. diameter and 30 ft. high, built, without centering, of mud which sets hard in the sun. The shells are so thin as to appear to be almost a structural absurdity : but it should be noted that they are formed to almost perfect parabolic curves, which eliminate all hoop tensions due to the weight of the material. Also they are most ingeniously ribbed on the outside in a manner which not only forms a convenient ladder for bare feet, but which also serves to strengthen, as well as to protect, the shell against local injury.

The foregoing theory is, strictly speaking, applicable only to thin shell domes. In thick domes numerous and complicated secondary stresses occur. The dominating fact is that for any given radius, any increase in the thickness beyond which is necessary to carry external loads does not increase the strength at all, the stresses per square inch remain the same.

To go on adding material, whether in a dome or girder, after enough has been provided to do the work, is not only expensive, it is dangerous, because of the added load on the bearings. A segmental or saucer dome supporting its own weight is a most particularly strong form of construction, and the complete hemispherical dome is nearly as strong, unless stone lanterns or heavy external loads are placed on the top, so that the thickness in unloaded domes necessary for stability bears no comparison to that necessary in the case of arches.

The loads, by no means inconsiderable, which will be supported by thin glass-domed lamp shades, inverted earthenware cups, and even by unbroken egg-shells in egg-cups, form instructive illustrations of the strength of thin domes even in brittle and unsuitable materials.

PART VI

COLUMNS AND SHORING

CHAPTER XXIX

COLUMNS

Impossibility of Defining the Exact Strength of Columns—Practical Approximations—Radius of Gyration—Ellipse of Inertia—Deduction of Radius of Gyration from Section Modulus, etc.—Calculation of Radius of Gyration of Compound Sections—Experiments on Model Columns—Ideal Column—Resiliency—Action of an Ideal Column under Load—Radius of Gyration a More Exact Criterion of Strength than Least Diameter.

IMPOSSIBILITY OF DEFINING THE EXACT STRENGTH OF COLUMNS.—All theoretical statements upon a subject are, or should be, merely the expression of actual results, and when the range of experimental research and the volume of practical experience is small, the formulæ and theoretical rules which investigators deduce from such experiments are apt to contain errors. As further experiments are made and additional experience is gathered, it is occasionally found that the earlier practical knowledge was incomplete and the rules which had been deduced from it were misleading.

Sometimes it is found that while the few earlier experiments pointed in one direction, yet this direction was only true for these particular results, and more complete experience—rightly viewed—indicated that the general laws governing the subject were quite different from what had been at first supposed. In studying the strength of columns the student must be prepared to find material differences between the conclusions drawn by earlier writers and those generally accepted at the present time. The earlier investigators found that short columns apparently failed by crushing, and long columns by bending; and they therefore divided columns up into classes, long and short, and assigned different fixed values to each class. Now we know that the two classes are identical and can be valued by the same laws. They also found that for simple sections the smallest diameter was apparently a vital dimension, and accordingly valued columns by the proportion between length and their least diameter. It is now recognised that a more correct criterion of relative strength is the radius of gyration—a somewhat more complicated dimension depending upon the shape of the section, and the way the material is disposed on each side of the neutral axis. In addition to these earlier misconceptions the subject is really incapable of exact definition. The ultimate strength of columns is a matter which cannot be fixed exactly, and all efforts to plot the results of experiments to any definite curve are bound to fail—because minute and quite immeasurable differences in the elasticity of the material will sensibly affect the results, which can only be expected to place themselves haphazard anywhere in the space between two curves representing an upper and a lower limit of strength. It is not suprising, therefore, that different authorities should have devised different formulæ correctly expressing some curve lying within this area, upon which curve some extensive range of results have happened to place themselves. Nor is it surprising that such formulæ are often very complicated. It is a curious trait of human character that many people will trust a formula that looks complicated and scientific and mistrust one which looks simple.

The student must, therefore, be prepared to find in use many different formulæ, and even to find writers of acknowledged and well-deserved reputation so embarrassed by the available wealth of standard rules as to advocate the calculation of columns by the weird process of applying a number of different formulæ and averaging the results.

PRACTICAL APPROXIMATIONS.—It will, however, be found that there is very little to choose between the different formulæ, and for practical design the simplest formula of all which can be adopted is one which represents a straight line drawn through that part of the doubtful area of the mapped out experiments which is most thickly occupied by results.

The method adopted in the following pages is to first examine the causes which tend to make the results uncertain—then to establish the limits of uncertainty, and lastly to plot curves showing how the various best known formulæ fall between those limits.

RADIUS OF GYRATION.—It was, as has been already stated, at one time customary to compare the relative stiffness of columns and stanchions of different sections by considering the relation existing between the least width of their cross section (d) and their length (l), viz., the ratio $\frac{l}{d}$.

The dimension of least width, however, was no criterion of the disposition of material in the cross section. It drew no distinction between the value of material advantageously placed at the outer edges of the cross section, as in a hollow column, and that placed at or near the centre in a solid section. Instead of *d*, a lineal dimension can be used which is deduced from the moment of inertia, and which is, therefore, proportional not only to the external dimensions of the cross section, but also to the disposition, advantageous or otherwise, of the material in it. This dimension is termed the Radius of Gyration (r), and is obtained by taking the square root of the moment of inertia of a section divided by its superficial area, or—

$$r^2 = \frac{I}{A}$$

$$r = \sqrt{\frac{I}{A}}$$

It is not easy to form a clear mental picture of the dimension radius of gyration. It is borrowed from the science of gyrating, viz., rotating bodies. The energy stored up in the particles of a rotating fly-wheel is found to depend upon the size and weight of those particles multiplied by the square of their distance from the centre of rotation. The sum total of those stored-up energies forms the "inertia" of the wheel, *i.e.*, its resistance to the starting of rotation, and is termed its moment of inertia (I). Such a fly-wheel can be imagined as flattened out into a broad but infinitely thin drum of the same area in cross section, and possessing the same moment of inertia against rotation round the same centre. *The radius of such a hypothetical drum would be the radius of gyration of the fly-wheel* (Fig. 184).

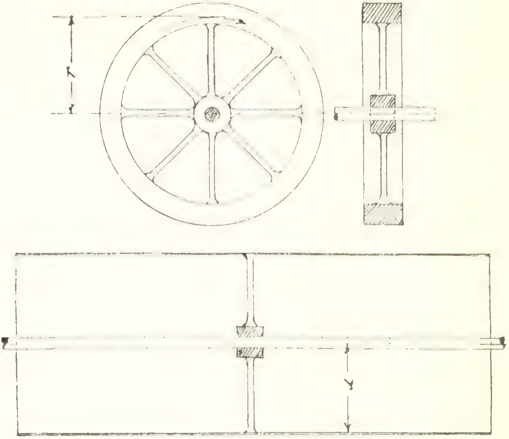


Fig. 184.

Similarly, two balls rotating on radial arms (Fig. 185) possess stored-up energy equal to the energy which has to be applied to cause them to start

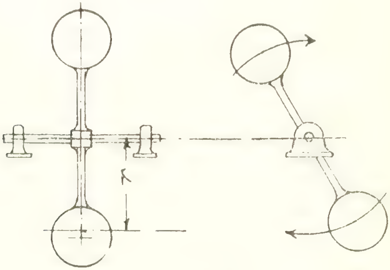


Fig. 185.

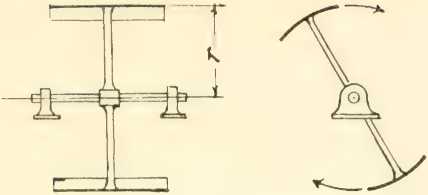


Fig. 186.

rotating, and the amount of this stored-up energy is their moment of inertia (I). They can be imagined as flattened out into thin plates (Fig. 186), and the distance from the centre at which those plates must be fixed upon the radial arms in order to possess the same moment of inertia as the balls should be the radius of gyration (r) of the balls. Instead of two balls, one can imagine a short length, say 1 in. long, of the rim of a fly-wheel fixed on opposite radial arms. Their cross section would then be similar to that of a girder, the radial arms forming the web as in Fig. 187. Such a *girder* would not, of course, rotate, but the moment of inertia of its cross section would be similarly calculated, viz., the sum of the area of each thin layer multiplied by the square of its distance from the neutral axis.

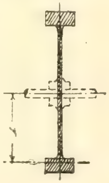


Fig. 187.

The cross section of the girder can, therefore, be similarly imagined as transformed into ideal conditions of efficiency with its whole area compressed into two infinitely thin layers or flanges, separated from each other by an infinitely thin web, and at such a distance (r) from the neutral axis midway between them *that the I of the ideal section is equal to the I of the section* *in its original form. That distance is termed its radius of gyration* (r).

The *strength* of such an ideal section as represented by the modulus $\frac{I}{y}$ would, of course, be greater than that of the section in its original form; because the strength of the whole section is limited by that of the part most severely tried under bending, *i.e.*, the fibres which lie at the extreme distance (y) from the neutral axis. This distance (y) in all practical sections must be greater than r. The whole area of the hypothetical ideal section being at one distance (r) from the neutral axis, the I of the section, both in its ideal and its original form, can be found with one calculation only, viz.:—

$$I = Ar^2.$$

The strength modulus Z of such an *ideal* section would be equally simple:—

$$Z = \frac{I}{y} = \frac{Ar^2}{r} = Ar,$$

which is the same as the area (a) of one flange or $\frac{A}{2}$ × the full depth $2r = d$ or $Z = ad$. The approximate statement $Z = ad$ is, therefore, only true for ideal sections.

The more closely the radius of gyration (r) of a section approaches the dimension y of the distance of the extreme fibre from the neutral axis, the more efficient and economical the section is.

In the uneconomical rectangular section with a large proportion of partially employed fibres near the neutral axis, the dimensions r and y are $\frac{d}{\sqrt{12}}$ and $\frac{d}{2}$

respectively. In standard rolled joists $\frac{d}{2}$ is approximately equal to half the depth between the flanges; and in plate and lattice girders, which more closely approach the lines of the ideal section, $\frac{d}{2}$ can without serious error be assumed to be equal to half the effective depth between the centres of gravity of the two flanges.

Ellipse of Inertia.—As there are an infinite number of directions in which a beam can be bent, there are an infinite number of strength moduli, and an infinite number of moments of inertia in its section, so also there must be an infinite number of corresponding radii of gyration. In all ordinary sections there is a greatest and a least radius of gyration corresponding to moments of inertia calculated along the directions of greatest and least strength. In the direction of greatest strength the area of each half of the section of the rolled joist in Fig. 188 may be imagined as compressed into an infinitely thin layer parallel to the flanges and to the horizontal neutral axis, and at a distance rx from it. In the direction of least strength this layer must be imagined as being

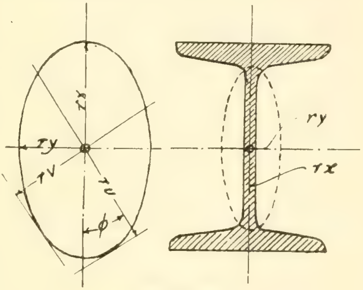


Fig. 188.

in a position at right angles to its previous position, viz., parallel to the web and to the vertical neutral axis and at a distance ry from it. Between these two extremes the imaginary infinitely thin layer can take up any number of positions at right angles to any number of different axes. Each position, however, would be tangential to the curve of an ellipse, of which $2\,rx$ and $2\,ry$ are the major and minor axes respectively. This greatly simplifies the calculation of the strength of any section in intermediate directions; for if the greatest and least radii of gyration rx and ry be known, then the length of any other radius ru inclined at an angle ϕ to say rx is given by the equation :—

$$(1) \qquad ru^2 = rx^2 - \sin^2\phi\,(rx^2 - ry^2),$$

and the corresponding radius rv at right angles to ru is given by the equation :—

$$(2) \qquad rv^2 = rx^2 - \cos^2\phi\,(rx^2 - ry^2).$$

The addition of (1) and (2) gives—

$$ru^2 + rv^2 = rx^2 + ry^2,$$

which, multiplied by the area of the section, gives—

$$Ix + Iy = Iu + Iv.$$

Therefore the sum of any two moments of inertia of a section about axes at right angles to each other is a constant quantity.

DEDUCTION OF RADIUS OF GYRATION FROM SECTION MODULUS, ETC.—It will be observed that if the radius of gyration and the area of a section be known and the distance y, its moment of inertia and its section modulus z can be ascertained from these particulars. Similarly, the radius of gyration or the moment of inertia can be obtained from the modulus Z thus:—

$$r^2 = \frac{I}{A} \quad Z = \frac{I}{y} \quad I = Zy,$$

$$r^2 = \frac{Zy}{A},$$

$$r = \sqrt{\frac{Zy}{A}}.$$

The greatest r of the 12×6 joist shown in Fig. 71, page 150, having an area of 12.946 sq. in., would be—

$$\sqrt{\frac{52.573 \times 6}{12.946}} = 4.936.$$

This is approximately half the depth between the flanges.

The corresponding approximate least radius of gyration would be that of a rectangle 6 in. deep.

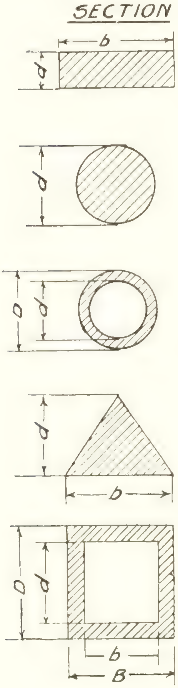
The I of any rectangle is $\frac{bd^3}{12}$ and its r is, therefore:—

$$\sqrt{\frac{bd^3}{12 \times bd}} = \sqrt{\frac{d^2}{12}} = \frac{d}{\sqrt{12}}, \text{ or roughly } \frac{d}{3.5}.$$

In this case $\frac{d}{3.5} = 1.7$, whereas the exact least r of the standard joist is 1.311.

APPROXIMATE RADIUS OF GYRATION OF STANDARD ROLLED JOISTS.—For British Standard sections a very close approximation of the least radius of gyration is $\frac{1}{4.75}$ of the least width, and of the greatest radius of gyration one-half of the depth between the flanges, *i.e.*, the full depth less the thickness of two flanges; the latter being about .2 to .22 of the width.

These approximations will be found extremely useful when the only particulars available are the dimensions, actual or assumed, of the cross section of a joist.



Least radius of gyration—

$$\frac{d}{\sqrt{12}} = 0.2887d.$$

$$\frac{d}{4}.$$

$$\frac{\sqrt{D^2 + d^2}}{4}.$$

$$\frac{d}{\sqrt{18}} = 0{\cdot}236d.$$

For equilateral triangles, 0.20396.

$$\sqrt{\frac{BD^3 - bd^3}{12(BD - bd)}}.$$

Fig. 189.

The radii of gyration of some simple sections is shown in Fig. 189. Those of more complex section can be found in any text-book, but they are generally deduced more easily from the section modulus or the moment of inertia.

CALCULATION OF RADIUS OF GYRATION OF COMPOUND SECTIONS.—Although it is seldom necessary in practice to calculate the I of a rolled section in order to arrive at the section modulus Z, it is very frequently necessary in the design of steel roof trusses to know the radius of gyration of compression members, which may be composed of two or three rolled sections riveted together.

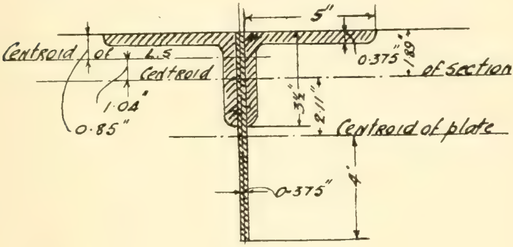


FIG. 190.

The labour of calculating the I of such members to arrive at the r by means of the equation $r = \sqrt{\frac{\mathrm{I}}{\mathrm{A}}}$ is greatly reduced by using British Standard sections, and referring to the published tables of the properties of such sections. Taking, for example, the built-up section shown in Fig. 190 for a principal rafter and composed of two 5-in. × $3\frac{1}{2}$-in. × $\frac{3}{8}$-in. standard angles, and an 8-in. × $\frac{3}{8}$-in. plate, the necessary calculations to arrive at the vital ratio $\frac{l}{r}$ are (neglecting rivet holes) as follows :—

Calculation of R *in a Vertical Direction.*

$$\text{Area of each L. S.} = 3.05.$$
$$\text{,,}\quad \text{plate} = 3.00.$$
$$\left.\begin{array}{l}\text{Distance of centroid of com-}\\ \text{pound section from top edge}\end{array}\right\} = \frac{(3.00 \times 4) + (6.1 \times .85)}{9.1}$$
$$= \frac{12 + 5.185}{9.1}$$
$$= 1.89.$$

Moment of Inertia of Entire Section.

$$\begin{array}{llr} & \left\{\begin{array}{l}\text{I of angles from tables}\\ \\ \text{I of web } \dfrac{\cdot 375 \times 8^3}{12}\end{array}\right. & \begin{array}{r}\left\{\begin{array}{r}3.09\\ 3.09\end{array}\right.\\ 16.00\end{array} \\ d & \left\{\begin{array}{l}3.05 \times 2 \times 1.04^2\\ 3.00 \times 2.11^2\end{array}\right. & \begin{array}{r}6.60\\ 13.36\end{array} \\ \hline & & \text{I} = 42.14\end{array}$$

Radius of Gyration of Whole Section.

$$r^2 = \frac{\text{I}}{a} = \frac{42.14}{9.1} = 4.62$$
$$r = \sqrt{4.62}$$
$$= 2.15.$$

EXPERIMENTS ON MODEL COLUMNS.—An extremely simple experiment throws a considerable amount of light upon the reason why columns fail erratically within certain limits. Take a fairly stout, straight, cane walking stick, with a round knob end, and press upon it with the hand, shifting the centre of pressure as necessary with the wrist to obtain a central load. It will soon be found that there is a certain direction where the pressure does not cause the stick to bend. If a slight lateral pressure be then applied with the knee the cane will bend, but will recover itself as soon as the lateral pressure is withdrawn. If vertical pressure in the same direction be increased, it will be found that the greater the vertical pressure the less will be the lateral push necessary to cause any given degree of bending, and the amount of bending will be found to depend not only upon the vertical pressure, but also upon the lateral push which starts the bending. At last a point will be reached when the least touch will be sufficient to cause bending. If the experiment be tried without any lateral pushing it will be found that under such a vertical load, if applied centrally and increasing in amount, the cane will up to a certain point offer a sensibly unyielding resistance to central or axial pressure, and when it gives way, it will give way all at once—the deflection or failure increasing *not with any increase in the vertical pressure* but simply with the continued application of the same pressure which caused the first collapse. If, however, the pressure be applied eccentrically or upon the head of a crutch-headed cane, then the behaviour is quite different. The bending then commences from the first application of the load and increases with the pressure, exactly as the deflection of an elastic girder increases with the load.

IDEAL COLUMNS.—The same experiment can be performed with advantage upon ordinary steel knitting needles, as in Fig. 191, but it will be found that much care and patience is required to get a needle to behave in the same manner as a round-headed cane under a central load. Even when it has been most accurately centred it will often still prefer to act in the same way as an eccentrically loaded stick, viz., deflecting or bending to an extent which varies with the load. The centrally loaded cane is an example of the *ideal* column in which there is no

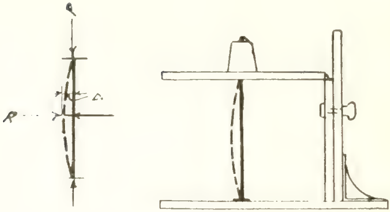


FIG. 101.

tendency towards bending, because the load is applied and kept by the delicate nervous mechanism of the hand and wrist exactly along the centre line.

An ideal column evidently has an inherent power of keeping itself straight under any load which is less than a certain amount. It may be bent by a lateral force, but as soon as that lateral force is removed it straightens itself up again.

RESILIENCY.—This power of straightening itself is called the *resiliency* of the strut. Resiliency, stiffness, or spring power is obviously closely allied to the stiffness which the strut would exhibit if it were acting as a girder. As a matter of fact the stiffness of the strut under an eccentric vertical force, or an axial force combined with a lateral force, depends upon its length, its cross section, and its material, just the same as the stiffness of a girder, under a transverse load, depends upon the same properties. If the resiliency or stiffness of a girder is weak, the girder deflects easily—if the resiliency of a strut is weak the strut bends easily.

ACTION OF AN IDEAL COLUMN UNDER LOAD.—When an ideal column is loaded exactly in the centre there is nothing for the resiliency to do—the load being exactly central, there is no tendency to bend, and no bending moment. Now consider what happens when a lateral force is applied ; a bending moment is set up—the resiliency of the column (acting as a beam) resists this bending moment—the material stretches on one side and compresses on the other, and a lateral deflection results. The load is now no longer central, but is acting at the leverage temporarily given to it by the deflection caused by the interfering lateral force. This makes a further call upon the resiliency, which is met, but at the cost of a further deflection, and of course a further advantage for the load and so on, each tiny additional deflection and additional bending moment growing smaller and smaller until at last they cease, and the load is supported by the stiffness of the bended strut. It will at once be seen that the greater the load is in proportion to the spring power of the strut, the larger is the first bend which it gives to the already bended strut, and the greater is the advantage which the load starts off with. If, on the other hand, the spring power or resiliency had been greater it would have had a larger advantage to start with, and would thus have been able to shorten the fight for mastery between itself and the load, and the ultimate deflection would have been smaller. Also if the independent lateral force had started the fight for mastery between the resiliency and the load by means of a bigger deflection, the load would have started with a bigger initial advantage and the resiliency would have been proportionately handicapped. The deflection of an ideal column under a vertical load plus a lateral force depends therefore on the relation between all three factors. Calling Δ the initial deflection caused by the lateral force, R the resiliency, and δ the ultimate deflection, then it can be shown that $\delta = \Delta\frac{\text{load}}{\text{R} - \text{load}}$.

If now the lateral force be removed from the loaded and bended column, the resiliency, being now relieved from the effort of resisting the interfering lateral force, is able to push the load up a little. This reduces the advantage gained by the load, which again reduces the deflection, and so on until the resiliency establishes its supremacy over the load and the column straightens out.

If the load be now increased, a point is reached when the smallest force will set the resilient force R and the load P at loggerheads. If P is now exactly equal to R then the column will remain just supporting the load in every position in which it may be bent to. If P is just the least shade greater than R, then the least touch is able to cause the column to break down or bend double.

RADIUS OF GYRATION A MORE CORRECT CRITERION OF STRENGTH THAN LEAST DIAMETER. —This leads us to a vital point in all column theory, viz., that a column cannot support a load greater than its own resilient force, and *the resiliency or stiffness of a column is thus at least one limit to its breaking load.* The resiliency of any column depends upon the way in which its material is distributed in its cross section, viz., upon its section modulus, the lineal criterion of which is its radius of gyration, and the true strength of columns does not depend upon their length ÷ their greatest depth or their least depth or their effective depth—it depends upon their length ÷ the expression which gives the true relationship of all three, viz., their radius of gyration r or $\frac{l}{r}$.

No column formulæ for columns of different sections are reliable unless they are based upon the ratio $\frac{l}{r}$.

The expression for the resiliency (R) of a column is too complicated to repay full investigation for the purposes of ordinary design. It is

$$R = EI\,\frac{9.87}{l^2}, \text{ or } R = EI\,\frac{\pi^2}{l^2}.$$

CHAPTER XXX

COLUMNS (*continued*)

Effect of Homogeneity of Material upon Column Strength—Limits of Column Strength—Fidler's Formula—Comparison of Different Column Formulæ—Straight Line Formula for Columns of Standard Steel—Additional Stress caused by Eccentric Loading—Straight Line Formula or Cast-Iron Columns—Straight Line Formula for Wood Columns.

EFFECT OF HOMOGENEITY OF MATERIAL UPON COLUMN STRENGTH.—The experiment of loading straight steel knitting needles as struts centred between iron plates is useful as showing how difficult it is even under such conditions to load steel columns centrally. In practice an ideal column in metal cannot be designed because it is commercially impossible to obtain cast iron, wrought iron, or steel so uniform in its structure that one side of a column or stanchion will be of exactly the same stiffness as the other side, as is practically the case with vegetable cane. It will readily be seen that if one side of a column be softer than the other, the most accurate centering is wasted, and practically all columns have to be considered as bows liable to bending, and therefore liable to failure as soon as ever the bending moment causes stresses in the external fibres greater than the material will stand. All columns are, in fact, beams standing on end, and are liable to failure as beams.

It will also be obvious that every column, however short and stout, is liable to increased stress on one side or the other if the load is not exactly central, *or* if its material is not exactly homogeneous; so that a formula that is true for long columns can be applied with equal truth to short ones.

LIMITS OF COLUMN STRENGTH.—When the British Standard specification is insisted upon, the limits of variation in the modulus of elasticity in commercial steel can be assumed with fair accuracy and it is possible to calculate what centric load will so far bend any column as to cause the elastic limit to be reached in the extreme fibres. This is shown by the lower line of Fig. 192. The formula for ascertaining this lower limit is somewhat long; its deduction, although interesting, is not easy, and it is not used in practical design.

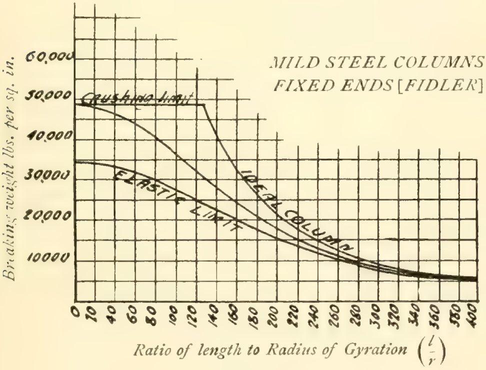


FIG. 192.

The failure of an ideal homogeneous column under an absolutely centric load would be caused either by direct crushing of the fibres in short stout columns, or by the load so nearly approaching the resilient force that the least touch would mean complete failure, *i.e.*, by the column being subjected to a centric load equal to the resiliency.

The strength of any given column may therefore be anywhere between the upper limit of a perfectly homogeneous ideal column and the lower limit of a column composed of metal which happens to be of extreme hardness on one side and extreme softness on the other.

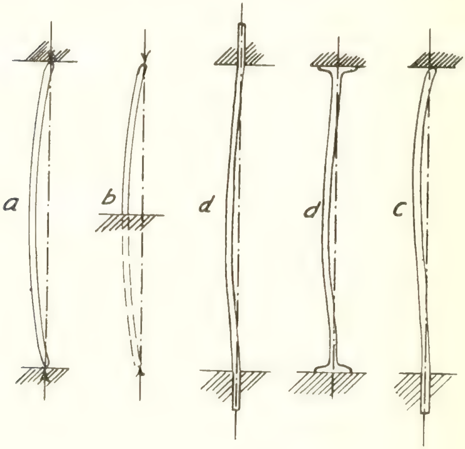


FIG. 193.

EFFECTS OF END FIXING.—Another very important point is the extent to which the ends are fixed. Fig. 193 shows the four usual methods of fixing usually assumed in column calculations: (*a*) Shows a column free to turn at either end, or pin connected. (*b*) Shows a column with one end fixed and one end quite free to move in any direction. This arrangement acts, as will be seen, just in the same way as the upper half of a pin-connected column of double its length. If its actual length is *l*, its effective length for calculation is 2*l*. (*c*) Shows a column with one end pin connected and one end fixed. (*d*) Shows a column with both ends fixed.

The effective length of a column at fixed both ends and subject to bending would approximate to half the length of a pin-connected column of the same length—the fixing of one end giving a value midway between the two. There are two ways of allowing for this difference. One is to use a formula which either has two values, one applicable to columns having ends absolutely free to turn, and another value for ends absolutely fixed and unable to turn, or, as both these conditions are ideal and cannot be obtained in practice, a value midway between the two. Another way is to use a formula representing the average strength of either a round ended column or one with fixed ends and then to treat each case on its merits. Where the fixing of both ends is particularly good and can be relied upon, then an effective length of $\frac{6}{10}$ of the round ended length can be taken; and where it can be partially relied upon, then an intermediate value can be assumed.

Fidler's Formula.—The true theory of columns centrally loaded, viz., that their strength lies anywhere between the limits of:—

1. The strength of an ideal column of perfectly homogeneous material;
2. The strength of a column of material as bad as could reasonably be expected,—

was first elucidated by Professor T. Claxton Fidler who deduced the following formula giving the approximate ultimate strength of an average column lying midway between the extreme limits as shown in the central curve of Fig. 192.

$$\text{Breaking load} = \frac{f+p-\sqrt{(f+p)^2-2.4fp}}{1.2}.$$

f = ultimate compressive strength = 26 tons per square inch for standard mild steel.

$p = \mathrm{E}\pi^2 \times \left(\frac{r}{0.6l}\right)^2$ for fixed ends.

$p = \mathrm{E}\pi^2 \times \left(\frac{r}{0.8l}\right)^2$ for one end fixed and one end hinged.

$p = \mathrm{E}\pi^2 \times \left(\frac{r}{l}\right)^2$. for both ends hinged.

E = modulus of elasticity—12,000 tons for standard steel.
$\pi = 3.1416$.
r = radius of gyration of column section in inches.
l = length of column in inches.

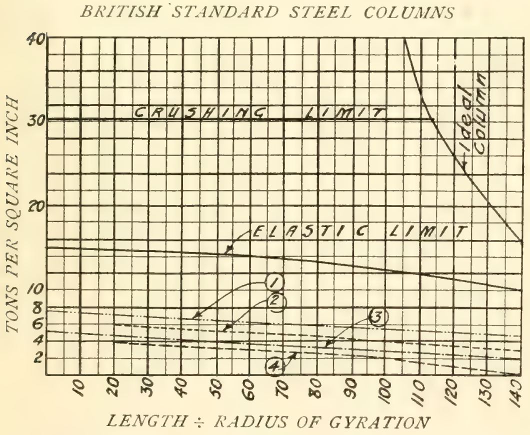
The deduction of this formula can be found in "Bridge Construction," by T. Claxton Fidler, or in a paper by the same author to be found in Vol. lxxxvi. of the *Proceedings of the Institution of Civil Engineers.*

When this formula is plotted to a curve and the results of very large numbers of actual tests are plotted on the same sheet, the path traced by the curve closely approximates to the position of the thickest constellations of results, so that the Fidler formula may be regarded as the most probably correct expression of the strength of the average practical column.

It must, however, be remembered that, as it is impossible to say how closely any particular column is going to approach the upper or the lower limit of strength—whether it is going to turn out a paragon, a reprobate, or merely respectable—there is no reason why one formula should be used instead of another so long as it expresses a value fairly within the extreme limits. If the Fidler formula be used the results would probably be more *likely* to be correct than by using any other—but as nothing can be certain and as the Fidler formula is a somewhat long one, there is no reason why it should be used in preference to a simpler form.

COMPARISON OF DIFFERENT COLUMN FORMULÆ.—When comparing different formulæ it is most necessary to ascertain—

1. What ultimate strength of steel is assumed by any given formula.



1. *S. L. Formula* $17500 - 57\frac{l}{r} = \frac{1}{4}$ *ultimate stress.*
2. *L.C.C. Gen. Powers Act*, 1909, *Fixed ends.*
3. *S.L. Formula* $12000 - 50\frac{l}{r}$
4. *L.C.C. Gen. Powers Act*, 1909, *Hinged ends.*

FIG. 194.

2. Whether the formula assumes fixed ends, round ends, or ends partially fixed.

In Fig. 194 are plotted for comparison the results of some well-known and largely used formulæ adapted to the properties of British Standard structural steel. The ratios of length to radius of gyration are not extended beyond 1 to 140, because a column of about seventy times its least dimension is quite as slender a structure as is likely to occur in buildings.

Even when these curves are set out to a very large scale one is compelled to omit certain well-known formulæ because the results are so nearly identical that the different curves become a confused mass of lines.

Above the curves of safe loads are shown a curve indicating the points at which columns of very irregular homogeneity would fail on account of the elastic limit being exceeded and the curves of failure of the ideal column either by direct crushing or by the resiliency being overloaded.

STRAIGHT LINE FORMULA FOR COLUMNS OF STANDARD STEEL.—The first inclined line below the elastic limit is that of the simple American straight line formula of

safe load in tons per square inch $= \dfrac{17{,}500 - 57\dfrac{l}{r}}{2240} =$ one-fourth ultimate strength.

In this formula $\dfrac{17{,}500}{2240} =$ one-fourth of the ultimate compressive stress per square inch on small specimens of the material.

This can be used by engineering firms who are not only sure of their design and of their workmanship, but also of their erectors and their materials. Its general adoption for building work is not to be recommended where these factors cannot be guaranteed. Below it are shown the curves of the safe loads allowed in the London County Council General Powers Act, 1909, for standard mild steel columns *with fixed ends* and *hinged ends.* The former is practically identical with the best known formula of all, viz., the Gordon-Rankine formula of $\frac{f}{1 - C\frac{l}{r}}$ or safe load in tons $= \frac{20}{1 - 3600\frac{l}{r}}$, with the constant C adjusted to the latest practice and used with a "sliding" factor of safety which increases as the column grows more slender—from 4 to 5 at an $\frac{l}{r}$ ratio of 140.

In the Gordon-Rankine formula, *f*, etc., are constants determined experimentally to express the variations in the ultimate strength caused by :—

f, variations in the ultimate strength in compression of small test specimens of the material.

C, variations in the modulus of elasticity of the material.

Below the line of the Gordon-Rankine formula are the L.C.C. safe loads for round ended columns, which are, of course, as improbable in practice as fixed ends.

Between the two falls the line of a modification of the straight line formula, viz., safe load in lbs. per square inch $= 12{,}000 - 50\frac{l}{r}$. This formula has the advantage of being very simple and easy to remember—it agrees with the best building practice—and it is midway between the L.C.C. limits for fixed ended and round ended columns respectively. As compared with the average failure point of the average practical column as represented by the Fidler formula, it has a factor of safety of about six. It may be taken as representing the average fixed ends formed in average good practice; but if for any reason the fixing of the ends should be doubtful, or if they are fixed to flexible supports, it would be better to multiply the length by some value between 1.0 and 1.5 to obtain the value of the corresponding round ended column.

ADDITIONAL STRESS CAUSED BY ECCENTRIC LOADING.—In *addition* to the bending moment caused by the tendency of a centrally loaded column to bend owing to the unequal stiffness of its fibres, it is necessary to consider the bending moments caused—

1. By eccentric loading.
2. By deflecting beams rigidly attached.

If, for instance, to take the simplest case, a girder rests unbolted on a bracket or cap, Fig. 195, its deflection under load causes it to transmit its load on to the *edge* of the bracket which may be some inches away from the centre line of the column. This causes a severe bending moment on the column, and can be dealt with by the ordinary formula $\frac{W}{a} \pm \frac{M}{Z}$.

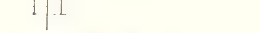


FIG. 195.

The maximum compression per square inch inflicted by load W acting at an eccentricity of x would be equivalent to that of a central load increased in the proportion of

$$\frac{W}{a} \text{ to } \frac{W}{a} + \frac{Wx}{Z},$$

a being the area of the column section in square inches, but as $Z = \frac{I}{y} = \frac{ar^2}{y}$ the ratio can be written

$$\frac{W}{a} : \frac{W}{a} + \frac{Wxy}{ar^2}.$$

The centric load equivalent to a load acting at an eccentricity of x would, therefore, equal $W \times \left(1 + \frac{xy}{r^2}\right)$. If, however, the girder were bolted to the top of the column the bending moment would probably be reduced; but its amount would be tedious to determine, and the results would be affected by variations in the flexibility of the bolts caused by varying tightness of the nuts. Provided the bolts were all screwed hard up, then if the proportion of girder and stanchion were both very slight it would be severe—if they were stout it would be very slight. In ordinary building practice, however, it is not possible to depend upon the rigidity of connections, and this tedious calculation is seldom made. To be on the safe side

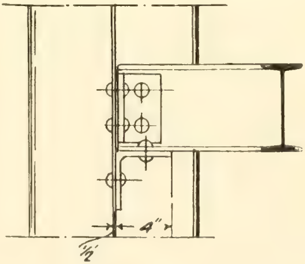


FIG. 196.

it is customary to assume even well bolted or riveted connections as acting at the centre of their bearing and calculate for this amount of eccentricity. Loose connections would, of course, act on the outside edges of their bearings.

Thus a joint secured to the $\frac{1}{2}$-in. web of a rolled stanchion by means of a 4-in. cleat, Fig. 196, would be taken as acting at an eccentricity of $\frac{4 \text{ in.}}{2} + \frac{\frac{1}{2} \text{ in.}}{2}$ or $2\frac{1}{4}$ in. If the area of the stanchion were, say, 21 sq. in., the reaction conveyed by the joist 20 tons, and the least section modulus 18, then the extreme fibre stress would be—

$$\frac{W}{a} \pm \frac{M}{Z} = \frac{20 \text{ tons}}{21 \text{ sq. in.}} \pm \frac{20 \text{ tons} \times 2\frac{1}{4} \text{ in. leverage}}{18}$$
$$= 0.95 \pm 3.75$$
$$= +4.7$$
$$\text{and} \quad -2.8,$$

i.e., the load would produce an extreme stress of 4.7 tons per square inch compression on one side, and 2.8 tons tension on the other, and its length divided by its radius of gyration should be not greater than, say, 40, that being the limit for slenderness for a loading of 4.7 tons per square inch.

CAST-IRON COLUMNS.—For cast iron a similar straight line formula of safe stress per square inch $p = 10{,}000 - 90\frac{l}{r}$ may be used up to an $\frac{l}{r}$ of 90, which is about the extreme practical limit of slenderness for cast iron.

WOOD COLUMNS.—Our knowledge of wood columns is very imperfect. In Fig. 197 are plotted two curves (3) and (4) from safe loads for timber columns suggested in Rivington's "Building Construction" as a practical mean average of various authorities.

A series of experiments, made by Mr Brereton, C.E., upon square columns of American yellow pine with ends adjusted as in ordinary practice, would appear,

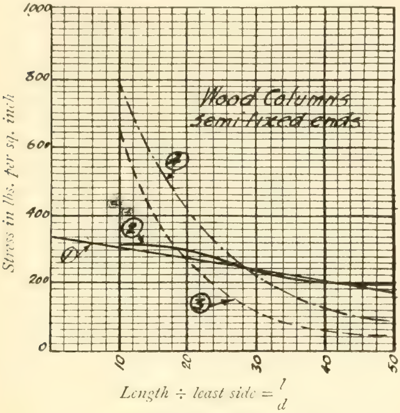


FIG. 197.

according to a table prepared by Mr Stoney, C.E., to yield somewhat different results.

These results are plotted in Fig. 197 (2) in the form of breaking load per square inch in lbs. ÷ 6.

A straight line formula which very closely approximates to these (1) would be

$$1.\ \left.\begin{array}{l}\text{Safe load in lbs. per square inch}\\ \text{up to 50 diameters long}\end{array}\right\} = 350 - 14\frac{l}{r}.$$

With rectangular sections r bears a constant ratio $\left(r = \frac{d}{\sqrt{12}} \text{ or } 0.2887d\right)$ to the length of least side d. So that for rectangular wood posts the formula can be more conveniently used in the form of

$$\left.\begin{array}{l}\text{Safe load in lbs. per square}\\ \quad\text{inch up to } \frac{l}{d} = 50\end{array}\right\} = 350 - 4\frac{l}{d}.$$

For circular wood posts $r = \frac{\text{radius (R)}}{2}$ and the formula (1) would in such cases be used in the form of $350 - 7\frac{l}{\text{R}}$, R being the radius of the cross section.

As there is very little practical difference in the modulus of elasticity of different woods there is no need to vary the above formula for oak or hard wood columns, but it should be remembered that there is no advantage in stiffness to be gained by substituting, say, oak for deal for a long slender wood column, but rather the reverse.

CHAPTER XXXI

SHORING

Shoring Timbers usually Determined by Expediency rather than by Calculation—Post and Needle Shoring—Flying Shoring—Horizontal Thrust on Shores—Raking Shores—Inclination of Sole Pieces.

SHORING TIMBERS USUALLY DETERMINED BY EXPEDIENCY RATHER THAN BY CALCULATION.—When shoring is necessary there is seldom any time to lose, and the dimensions of the timbers used are more frequently determined by considerations of expediency rather than by calculation.

It is, however, very desirable to know the direction and extent of the forces acting upon shoring timber, especially as there are some fairly established fallacies in existence with regard to the stresses in raking shores.

Shoring can be divided into the broad classes of :—

(*a*) Vertical or needle shoring.
(*b*) Horizontal or flying shoring.
(*c*) Inclined or raking shoring.

POST AND NEEDLE SHORING.—A typical case of needle shoring would be the temporary support given to the upper part of a wall when it is desired to cut away the lower part. Such support would be obtained by cutting holes through the wall a few feet apart above the level of the top of the proposed opening and threading stout wooden "needles" of timber through these holes—the ends being supported on wooden posts (Fig. 198). In this, as in every other case of shoring, it is very necessary to remember the importance of providing pairs of folding wedges either at the top or bottom of the posts to enable the needle or supporting timber to be forced tight up against the load which it is designed to support. It is also obvious that the pressure from the feet of the posts should be distributed by stout sole pieces and transmitted to a solid foundation.

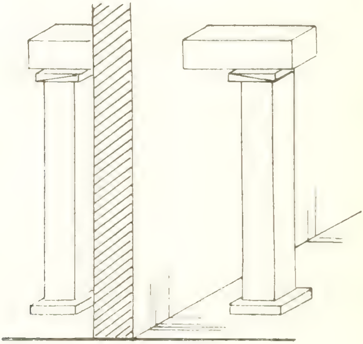


FIG. 198.

Posts and needles are columns and girders of the simplest description, and they are both kept as short as they conveniently can be. The needle is generally considered as a beam centrally loaded, but if it is supporting thick masonry the amount of distribution might well be taken into account.

Brickwork or bonded masonry will support itself over considerable spans if the end abutments be sufficient: so that the considerations which determine how many needles should be put in would be—(1) What is the weight of the wall and the loads on it per foot run, and how many feet run of wall can be safely taken by the needle and posts of the sizes available? (2) Is the mortar of the masonry good enough to hold up the portion below the concealed arch of the bonding over the distance from needle fixed by (1)? Taking the case of a warehouse of several floors with a dividing wall on each floor. It is desired to cut away this wall on the ground floor for a width of, say, 20 ft. × 10 ft., and to carry it on a girder, Fig. 199. The weight of the wall itself and the weights transmitted to it by the various upper floors is calculated and found to amount to, say, 40 cwt. per foot run. Timber 10 in. square is available—how many needles should be put in?

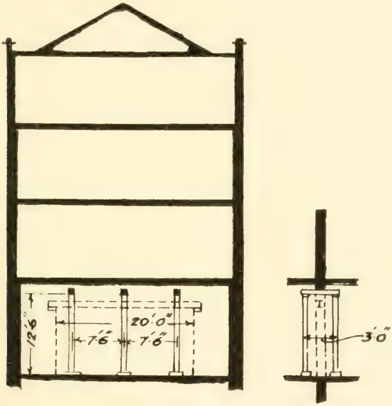


FIG. 199.

Two would give a span of 10 ft. to be supported by each needle. Three would give 7 ft. 6 in., and five 5 ft., and six 4 ft. Needles 4 or 5 ft. apart would be rather close, so that 7 ft. 6 in. would be tried. The posts would be, say, 12 ft. long, which, allowing for two 3-in. sole pieces, would give a distance of 2 ft. 6 in. below the bottom of the needles in which to manipulate the girder—say posts of thirteen times their least dimension $\frac{l}{d} = 13$. They would be flat bedded and more or less fixed at their ends. Their safe load per square inch would be

$$350 - 4\frac{l}{d} \text{ lbs.} = 350 - 52 = 298,$$

and they would be regarded as struts capable of taking, say, 0.14 ton per square inch, or 20 tons each. Timbers 10 in. square would then be amply strong enough whatever the number of needles, because the whole length of 20 ft. only weighs 40 tons and each needle rests on two posts.

The next point to consider is the length of the needle. For convenience in working this should be, say, 5 ft. A distance of 7 ft. 6 in. from needle to needle would give a load of 15 tons on a beam of 5 ft. span. The handy formula of $\frac{bd^2}{S}$ for safe distributed loads assumes a factor of safety of 6.6, but with timber used for temporary purposes this factor of safety can be reduced. A factor of safety of two is necessary to keep within the elastic limit, and as we have assumed an absolutely central load which is really more or less distributed we can safely work to a factor of safety of from 3 to $3\frac{1}{2}$. Assuming a factor of safety of 3.3 the handy formula would read $\frac{2bd^2}{S}$ for a distributed load, or $\frac{2bd^2}{2S} = \frac{bd^2}{S}$ for a central oad. The safe load of a 10-in. × 10-in. needle on a 5-ft. span would be, therefore, $\frac{10 \times 10 \times 10}{5}$ cwt., or 10 tons. A distance of 7 ft. 6 in. between the needles, giving a load of 15 tons, would be, therefore, too much for a single needle, but that load would be taken by two needles side by side, or better still, one over the other. It would be doubtful whether the triangle of masonry or brickwork which the bonding would leave free to drop out between the needles would be safely held up by the tenacity of the mortar during the process of cutting away, and probably needles 5 ft. apart would for this reason be adopted. It is obvious that in a case like this the wall would be relieved of the floor loads by a system of secondary strutting from floor to floor ; but this is not always practicable if the upper floors are to be kept in use and undisturbed during the alteration.

FLYING SHORING.—Examples of flying shoring are very common in the streets of towns. When a building is taken down the adjoining buildings on each side are deprived of any support which they may have been receiving from it. It may not be known for certain that they have been leaning on the removed building, but there is generally the chance that they may have been doing so.

The place of the removed building is, therefore, taken by horizontal braced struts, as Fig. 200, about 10 to 15 ft. apart, with the horizontal member at about three-fourths the full height of the buildings.

A stout wall plate, say 9 in. × 3 in., with stout cleats to take the ends of the secondary struts, is first secured to each wall to be supported, a half-header being cut out of the brickwork and rebated timber toggles passed through the centre of the wall plates, and further secured by cleats. The horizontal shore is then cut a little shorter than the distance between the wall plates, sole pieces are spiked on top and bottom, and it is hoisted on to the cleated toggles and wedged tight by two pairs of folding wedges.

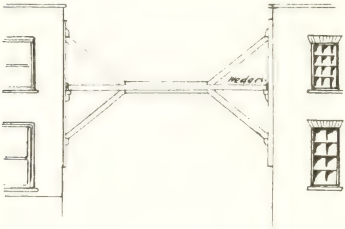


Fig. 200.

The secondary struts are then accurately cut to a tight fit and driven home and spiked without wedging.

Horizontal flying shores are usually limited to spans of about 30 ft. because of the difficulty of obtaining timber for the main shore of a greater length than that, and because of the great difficulty of scarfing and fishing timber to obtain anything like the strength of unjointed timber. Also timber over 10 in. square is difficult to obtain, and a timber column more slender than 30 ft. × 10 in. is not very efficient as a strut, and in addition the strength of a long heavy horizontal strut would be largely occupied by supporting its own weight, which weight would also be hanging on to old walls. Although pitch pine can be obtained 50 ft. and 60 ft. long, raking shores would probably be cheaper and stronger, or if raking shores were absolutely impossible, then probably a lattice steel strut of box section.

HORIZONTAL THRUST ON SHORES.—The pressure that a shore is going to be called upon to take depends upon whether the walls supported are overhanging or not. If they are plumb and the mission of the shore is merely to prevent any tendency of the wall to squeeze out a temporarily disturbed foundation on the exposed side, then the overturning moment would be the weight of the wall W and any floors, roof, etc., that it may carry multiplied by one-half the thickness of its base T. The leverage of the shore on resisting this tendency, would be A B, its vertical distance from the base of the wall.

$$\text{Thrust} = \frac{W \times \frac{1}{2}T}{AB} \text{ or } \frac{WT}{2AB}.$$

Although both walls might have a tendency to fall inwards, the thrust on a flying shore would be only the greater tendency of the two. If either wall already overhangs, then the extent of overhang must be added to $\frac{T}{2}$. If there is a distinct thrust on either side, say from an insufficiently tied roof, this must be estimated and added. The length of a flying shore as a column is its full span, because the secondary struts afford no relief against lateral bending. Lateral struts can be put in, but they are not usual.

As a rough guide to the sizes of flying shores it may be assumed that for spans of 15 or 30 ft. timbers of 6 in. × 6 in. to 10 in. × 10 in. are used. Up to 20 ft. span 6 in. × 6 in. would probably be used. From 20 ft. to 25 ft., 6 in. × 6 in. or 8 in. × 8 in.; 25 ft. to 30 ft., 8 in. × 8 in. or 10 in. × 10 in.

If an unequal timber like 8 in. × 6 in. is used it should be laid flat, so that its smallest dimension will be stiffened by the secondary strut.

RAKING SHORES.—These also are a familiar sight in town streets. They consist essentially of one or more inclined shores, Fig. 201, bedded on a solid sole piece at the foot, with the head wedged tightly up against a stout rebated pin or toggle passing through a stout wall plate and into a hole in the supported wall made by cutting out a half header, Fig. 202; a secondary strut being generally introduced to take up the tendency of the shore to bend under its own weight.

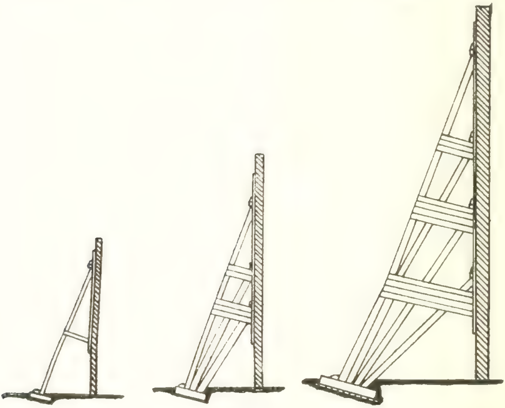


FIG. 201.

The leverage of such an inclined shore against any tendency of a wall to overturn is the vertical height of the head of the shore above the base of the wall. But the shore itself can only push or react upwards in the direction of its own inclination, and this capacity for inclined reaction can only be utilised to secure a horizontal reaction by loading the head of the shore.

This is effected by making the head of the shore bear against the toggle going into the wall and with a sufficient weight of wall above it to prevent the brickwork or masonry from bursting upwards. The amount of useful horizontal thrust possible depends upon the vertical weight, and can be ascertained graphically by setting up vertically to scale the weight of the brickwork above the head of the shore (W B, Fig. 203), and from the top of this line drawing a horizontal (R) to meet the direction of the inclined shore produced. The length of the inclined line cut

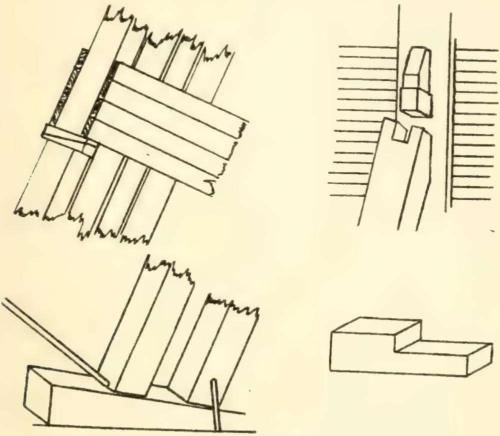


FIG. 202.

off will be the maximum compression which can be obtained in this shore without lifting up the supporting brickwork ; and the length of the horizontal line gives the maximum horizontal thrust which can be exerted on the wall by wedging up without bringing about the same result.

In addition, it is necessary to consider the weight of the shore itself, which partly rests on the wall plate and is resisted by a horizontal reaction against it. The extent of this horizontal reaction can be ascertained by taking moments about the foot of the shore, Fig. 203. The weight of shore W acts at half the horizontal distance B C between the wall and the foot and sets up a clockwise moment of $W \times \frac{BC}{2}$. The horizontal reaction r, due to the weight of the shore, acts at a leverage of the vertical height A B of the head above the foot and causes an anti-clockwise moment of $r \times AB$.

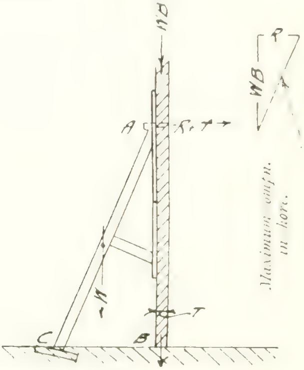


FIG. 203.

From these two conditions r can be calculated thus :—

$$r \times \mathrm{AB} = \mathrm{W} \times \frac{\mathrm{BC}}{2},$$

$$= r\frac{\mathrm{WBC}}{2\mathrm{AB}}.$$

But this small reaction r is somewhat modified by the action of the secondary strut.

The total reaction due to the weight of the shore r plus the main reaction from the wedging up R, or $\mathrm{R} + r$, gives the total reaction possible at the head of any single shore, and the leverage of $\mathrm{R} + r$ as against any overturning moment or moments to which the wall may be exposed, whether due to lateral thrusts or due to the weakness of its foundations, is the vertical height of the head of the shore above the base of the wall.

The tendency of the shore to bend under its own weight is, of course, resisted by the secondary strut.

INCLINATION OF SOLE PIECES.—It is sometimes suggested that the reactions due (1) to wedging up and (2) to the weight of the shore cause the total reaction to act along some imaginary line steeper than the inclination of the shore, and that the sole piece should be placed at right angles to this imaginary line. This is quite incorrect. The sole piece is laid at an inclination which will wedge up the shore tighter and tighter against the wall plate as it is levered further and further along by a crowbar, and has nothing whatever to do with the total reaction.

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